Current Version: July 1999

Accounting Information, Capital Investment Decisions, and Equity Valuation: Theory and Empirical Implications

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1. Introduction

This paper develops a theoretical model to reexamine the roles of earnings and book value for equity valuation and to explore cross-sectional differences in the properties of the valuation function. Ohlson [1995] and Feltham and Ohlson [1995; 1996] show that, assuming the clean-surplus relation and a specific linear information dynamic, equity value can be represented as a linear function of earnings and book value. However, recent empirical evidence indicates linear valuation functions do not fully capture the effects of earnings and book value on equity value. For instance, Burgstahler and Dichev [1997] find the valuation impact of either earnings or book value depends on the levels of these two variables, and the valuation function is convex, not linear; similarly, Hayn [1995] and Collins et al. [1999] report that the valuation effect of earnings is asymmetric between positive and negative earnings. Also, as Collins et al. [1999] show, the relative and the incremental importance of earnings versus book value in explaining equity value vary with the level of earnings. In this paper, I combine the valuation approach of Ohlson [1995] and Feltham and Ohlson [1995; 1996] with capital investment decisions to develop a model that reconciles these and other empirical results. I offer insights into the relation between accounting variables and equity value; in particular, I comment on cross-sectional differences in the behavior of the valuation function.

My model specifies a setting in which a firm may choose to expand its operations when it is sufficiently profitable and to discontinue them when sufficiently unprofitable. Valuation requires first forming beliefs about investment activity based on current operating efficiency and then valuing the cash flows to be produced from operations. This approach is in contrast to Feltham and Ohlson [1996] and other studies where the firm's intertemporal investment is assumed to follow a predetermined linear stochastic process and accounting information plays no explicit role in determining this process. In my model, intertemporal investment cannot be characterized by a single linear stochastic process because the chosen investment strategy depends on the observed information. With endogenous investment decisions, equity value becomes a non-linear function of accounting variables.

Equity value is derived initially in terms of asset stock and operating efficiency. With simplifications, equity value is shown to equal the expected value from maintaining the present course of operations plus the value of the option to expand or contract the scale of operations. Value depends on operating efficiency because efficiency measures the firm's ability to generate cash flows from assets, thereby indicating the desirability of investing or disinvesting. I then derive equity value in terms of accounting variables, conditional on three accounting rules: historical cost valuation, the clean-surplus relation, and a conservative (therefore biased) depreciation policy.

With conservative accounting, equity value is a function of both accounting variables and the bias in measuring them. If the biases have little influence on valuation, the model yields the following predictions. Holding book value constant, equity value increases with earnings for all firms, but, given earnings, equity value can be either increasing in, decreasing in, or insensitive to book value, depending on efficiency and growth potential. In general, equity value is convex in both earnings and book value, particularly for low efficiency firms and for high efficiency firms with growth opportunities. However, for firms expected to remain in a steady state, equity value reduces to a linear function of earnings, with book value having little incremental explanatory power. The model also implies that the relative importance of earnings versus book value in explaining equity value varies across firms that differ in efficiency and growth potential.

In my model, accounting conservatism causes book value to understate true asset values, which increases the valuation impact of book value. On the other hand, conservatism may either increase or reduce the marginal valuation impact of earnings because accounting earnings may understate or overstate true economic earnings, depending on past changes in investment scale.

Some implications of my model are consistent with existing empirical evidence. Examples include Burgstahler and Dichev [1997] on convexity and the positive effects of earnings; Burgstahler and Dichev [1997] on the non-monotonic effects of book value; and Collins et al. [1999] on the relative explanatory power of earnings versus book value. My model also explains why inverse associations are expected between stock prices and negative earnings, thus reconciling the empirical findings of Jan and Ou [1995], Hayn [1995] and Collins et al. [1999]. Several other predictions have been tested only indirectly or not at all.

In related research, Burgstahler and Dichev [1997] also predict convexity of equity value in earnings and book value, given an option to adapt firm resources to alternative use.[1] Burgstahler and Dichev do not develop a formal model; they are concerned mostly with empirical analysis. By developing a formal valuation model, I am able to examine more broadly the valuation function and its cross-sectional differences, and to generate a number of implications that are broadly consistent with empirical findings.

The rest of the paper is organized as follows. Section 2 describes the setting. Section 3 derives equity value in terms of fundamental information variables. Section 4 develops an accounting-based valuation model. Section 5 examines cross-sectional differences in the behavior of the valuation function and compares the model's predictions with existing empirical results. Section 6 explains the empirical result of inverse associations between stock prices and negative earnings. Section 7 develops additional implications of the model. Conclusions are provided in Section 8.

2. Description of the Problem

Consider a firm that operates in a multi-period setting. At date $\tau + 1, \tau \in \{0, 1, ...\}$, the firm receives a cash flow $cr_{\tau+1}$, produced by the assets at date τ, and determined by both the asset stock at date τ (denoted as_τ) and the firm's operating efficiency in period $\tau + 1$ (denoted $\tilde{\kappa}_{\tau+1}$) according to the following technology,

$$\tilde{cr}_{\tau+1} = \tilde{\kappa}_{\tau+1} as_\tau. \tag{1}$$

Operating efficiency follows a random walk,[2]

$$\tilde{\kappa}_{\tau+1} = \kappa_\tau + \tilde{\nu}_{\tau+1}, \tag{2}$$

where κ_τ is the efficiency for period τ, known at date τ, and $\tilde{\nu}_{\tau+1}$ is a zero-mean disturbance term that cannot be predicted on or before date τ. The stock of assets in place diminishes over time; new investment is required to replenish the stock. The total asset stock at date τ is determined by the date $\tau - 1$ stock level and the date τ cash investment, ci_τ, as follows,

$$as_\tau = \gamma as_{\tau-1} + ci_\tau, \tag{3}$$

where γ, $0 < \gamma < 1$, is a parameter that represents the durability of the assets.[3]

Applying (3) recursively yields

$$as_\tau = \sum_{s=0}^{\tau} \gamma^{\tau-s} ci_s. \tag{4}$$

Equations (1), (2) and (3) together imply that each dollar of cash invested at date τ is *expected* to produce a series of cash receipts, κ_τ, $\gamma\kappa_\tau$, $\gamma^2\kappa_\tau$, ..., at dates $\tau+1$, $\tau+2$, $\tau+3$, ..., respectively. The present value of this series is $\kappa_\tau/(R-\gamma)$, where R equals one plus the risk-free rate of return per period. If q_τ is the internal rate of return on cash investment at τ, it can be shown that

$$q_\tau = \kappa_\tau - (1-\gamma). \tag{5}$$

As q_τ differs from κ_τ only by a constant, $1-\gamma$, operating efficiency is equivalently represented by q_τ, and the time-series process for q_τ is the same as that for κ_τ, that is, $\tilde{q}_{\tau+1} = q_\tau + \tilde{\nu}_{\tau+1}$.

At date τ, the net cash flow from operations is $c_\tau \equiv cr_\tau - ci_\tau$. Following Feltham and Ohlson [1996], I assume that net cash flow is paid out as dividends. Let V_τ be the (ex-dividend) equity value at date τ. Then, in a risk-neutral world, $V_\tau \equiv \sum_{s=1}^{\infty} E_\tau[\tilde{c}_{\tau+s}]/R^s$.

The valuation problem in its general form involves investment decisions at different dates. To keep the derivation tractable, I simplify the structure of investment opportunities. Relative to date t, an arbitrarily chosen valuation point, all future investment opportunities are summarized by a set of choices at date t+1. The specific assumptions are as follows.

At date t, the firm can either discontinue operations or maintain them at the same scale as at date t-1. As the former option makes the problem degenerate, the latter is assumed, in which case $ci_t = (1-\gamma)as_{t-1}$ and $as_t = as_{t-1}$.[4]

At date t+1, the firm is faced with three alternative scenarios: (i) discontinuation of operations, in which case the value of the assets depends on the asset stock, and is assumed to equal $(1-c_d)as_t$, where $0 < c_d < 1$ is the cost of discontinuation; (ii) continuation at the same scale of operations as at date t, in which case cash investment required at date $t+1$ is $ci_{t+1} = (1-\gamma)as_t$; and (iii) expansion, in which case the asset stock is expanded to $as_{t+1} = as_t + G$, where G represents the growth potential,[5] and cash investment required at t+1 is $ci_{t+1} = (1-\gamma)as_t + G$.

To keep the analysis tractable, investment problems beyond date t+1 are suppressed. From date t+2 and onwards, the firm is assumed to maintain the operating scale set at date t+1. Thus, $ci_{t+s+1} = (1-\gamma)as_{t+s} = (1-\gamma)as_{t+1}, \forall s \geq 1$.

3. Determination of Equity Value at Date t

This section derives equity value at date t in terms of fundamental economic events. Valuation requires projections of future cash flows, which in turn depend on investment decisions. Thus, I first characterize investment behavior at t+1. Equity values in the three scenarios are compared to find the best course of action.

Scenario (i): Equity value at t+1 if operations are discontinued, denoted $V_{t+1}(i)$, is the discontinuation value of the assets,

$$V_{t+1}(i) = (1 - c_d)as_{t+1} = (1 - c_d)\gamma as_t. \tag{6}$$

Scenario (ii): If the firm continues operations at t+1 at the same scale as at t, $ci_{t+1+s} = (1-\gamma)as_{t+s}, \forall s \geq 0$. The expected net cash flow at date $t+s+1, s \geq 1$, is

$$E_{t+1}[cr_{t+s+1} - ci_{t+s+1}] = E_{t+1}[\tilde{\kappa}_{t+s+1}as_t - (1-\gamma)as_t] = q_{t+1}as_t. \tag{7}$$

And equity value at t+1 after the cash investment, denoted $V_{t+1}(ii)$, equals

$$V_{t+1}(ii) = \sum_{s=1}^{\infty} \frac{E_{t+1}[cr_{t+1+s} - ci_{t+1+s}]}{R^s} = \sum_{s=1}^{\infty} \frac{q_{t+1} \cdot as_t}{R^s} = \frac{q_{t+1}as_t}{R-1} = \frac{x^E_{t+1}}{R-1}, \tag{8}$$

where

$$x^E_{t+1} \equiv q_{t+1} \cdot as_t = cr_{t+1} - (1-\gamma)as_t \tag{9}$$

measures the net value creation in period t+1, referred to as "economic earnings".

Scenario (iii): If the firm expands operations at t+1, equity value at t+1 after cash investment, denoted $V_{t+1}(iii)$, is derived as in scenario (ii):

$$V_{t+1}(iii) = \frac{q_{t+1}(as_t + G)}{R-1} = \frac{x^E_{t+1} + q_{t+1}G}{R-1}. \tag{10}$$

Let $\hat{V}_{t+1}$ be equity value at date t+1 *before* cash investment. Then,

$$\hat{V}_{t+1} = max\{V_{t+1}(i), [V_{t+1}(ii) - (1-\gamma)as_t], [V_{t+1}(iii) - (1-\gamma)as_t - G]\}. \quad (11)$$

If scenario (ii) is the base scenario, the incremental benefit of choosing scenario (i) is $V_{t+1}(i) - [V_{t+1}(ii) - (1-\gamma)as_t] = (1-\gamma c_d)as_t - q_{t+1}as_t/(R-1)$. Scenario (i) is preferred to scenario (ii) if and only if $V_{t+1}(i) - [V_{t+1}(ii) - (1-\gamma)as_t] > 0$, or

$$q_{t+1} < q_d^* \equiv (1-\gamma c_d)(R-1). \quad (12)$$

Also, under condition (12), the firm prefers scenario (i) to scenario (iii).

Similarly, relative to scenario (ii), the incremental benefit of scenario (iii) is $[V_{t+1}(iii) - (1-\gamma)as_t - G] - [V_{t+1}(ii) - (1-\gamma)as_t] = q_{t+1}G/(R-1) - G$. Thus, the firm prefers scenario (iii) to scenario (ii) (and also to scenario i) if and only if

$$q_{t+1} > q_e^* \equiv R - 1. \quad (13)$$

To summarize, the investment decision at t+1 is: discontinuation if $q_{t+1} < q_d^*$, continuation at the same scale if $q_d^* \leq q_{t+1} \leq q_e^*$, and expansion if $q_{t+1} > q_e^*$.

Turning now to valuation, equity value at date t equals

$$V_t = E_t[\frac{cr_{t+1} - ci_{t+1} + V_{t+1}}{R}] = E_t[\frac{cr_{t+1} + \hat{V}_{t+1}}{R}]. \quad (14)$$

Based on the investment criterion derived above,

$$\begin{aligned} V_t &= E_t[\frac{cr_{t+1} + [V_{t+1}(ii) - (1-\gamma)as_t]}{R}] + E_t[\frac{\max\{0, V_{t+1}(i) - [V_{t+1}(ii) - (1-\gamma)as_t]\}}{R}] \\ &\quad + E_t[\frac{\max\{0, [V_{t+1}(iii) - (1-\gamma)as_t - G] - [V_{t+1}(ii) - (1-\gamma)as_t]\}}{R}] \\ &= E_t[\frac{q_{t+1}as_t + V_{t+1}(ii)}{R}] + \frac{1}{R}E_t[\max\{0, (1-\gamma c_d)as_t - \frac{q_{t+1}as_t}{R-1}\}] \\ &\quad + \frac{1}{R}E_t[\max\{0, \frac{q_{t+1}G}{R-1} - G\}]. \end{aligned} \quad (15)$$

Replacing $V_{t+1}(ii)$ by (8) and simplifying yield

$$V_t = \frac{1}{R-1}x_t^E + \mathcal{P}_d(q_t) \cdot as_t + \mathcal{C}_e(q_t) \cdot G, \tag{16}$$

where $\mathcal{P}_d(q_t) \equiv \frac{1}{R(R-1)} \int_{\underline{\nu}}^{q_d^*-q_t} [q_d^* - q_t - \tilde{\nu}_{t+1}] f(\tilde{\nu}_{t+1})\, d\tilde{\nu}_{t+1}$, $\mathcal{C}_e(q_t) \equiv \frac{1}{R(R-1)} \int_{q_e^*-q_t}^{\bar{\nu}} [q_t + \tilde{\nu}_{t+1} - q_e^*] f(\tilde{\nu}_{t+1})\, d\tilde{\nu}_{t+1}$, and $f(\tilde{\nu}_{t+1})$ is the probability density function of $\tilde{\nu}_{t+1} \in [\underline{\nu}, \bar{\nu}]$ with $\int_{\underline{\nu}}^{\bar{\nu}} \tilde{\nu} f(\tilde{\nu}) d\tilde{\nu} = 0$.

According to (16), equity value consists of the expected value from maintaining current operations, $x_t^E/(R-1)$, plus the value of the (put) option to discontinue operations at date $t+1$, $\mathcal{P}_d(\cdot)as_t$, and value of the (call) option to expand operations at date $t+1$, $\mathcal{C}_e(\cdot)G$.[6]

4. *Accounting-Based Valuation*

Equation (16) relates equity value to current operations as captured by (as_t, q_t). In practice, accounting is typically used to measure both asset stock (as_t) and efficiency (q_t). This section introduces accounting rules and establishes an accounting-based valuation equation.

4.1 ACCOUNTING RULES AND ACCOUNTING VARIABLES

Assuming historical cost valuation, then the book value of assets at the initial point (date 0), B_0, equals ci_0. (I assume all-equity financing, so the book value of assets equals the book value of equity.)

Depreciation policy is assumed to satisfy the clean surplus relation,

$$B_{\tau+1} = B_\tau - dep_{\tau+1} + ci_{\tau+1}, \tag{17}$$

where B_τ is the book value of assets, net of accumulated depreciation, at date τ and

dep_τ is depreciation expense for period τ. Depreciation policy is also assumed to be conservative; as in Feltham and Ohlson [1996], depreciation has the following form,

$$dep_\tau = (1 - \delta)B_{\tau-1}, \tag{18}$$

where $0 < \delta < \gamma$. Thus, over time, depreciation expense is recognized more quickly than true economic depreciation.

Equations (17) and (18) imply

$$B_{\tau+1} = \delta B_\tau + ci_{\tau+1}. \tag{19}$$

Applying (19) recursively yields

$$B_\tau = \sum_{s=0}^{\tau} \delta^{\tau-s} ci_s. \tag{20}$$

Comparing (20) with (4), $B_\tau < as_\tau$, given $\delta < \gamma$. That is, with conservative accounting, book value always *understates* the asset stock (except at date 0 when the two are equal). Let $u_\tau = as_\tau - B_\tau$ be the bias of book value in measuring the asset stock at τ. Then

$$u_\tau \equiv \sum_{s=0}^{\tau} (\gamma^{\tau-s} - \delta^{\tau-s}) ci_s. \tag{21}$$

The extent of bias in book value depends on the sequence of past investments, $\{ci_0, ci_1, ..., ci_{\tau-1}, ci_\tau\}$, and the degree of accounting conservatism (i.e., parameter δ relative to γ). While u_τ is influenced by all past investments, the weighting placed on a particular asset, $\gamma^{\tau-s} - \delta^{\tau-s}$, depends on its age, $\tau - s$. Generally speaking, the most recent investments have the greatest impact on u_τ.[7]

Given the accounting rules and book value, period τ earnings are

$$x_\tau \equiv cr_\tau - dep_\tau = cr_\tau - (1-\delta)B_{\tau-1}. \tag{22}$$

Based on (4), (9), (20) and (22), the relation between economic earnings and accounting earnings is $x_\tau^E = x_\tau + \Delta u_\tau$, where

$$\Delta u_\tau \equiv u_\tau - u_{\tau-1} = \sum_{s=1}^{\tau}(\gamma^{\tau-s} - \delta^{\tau-s})[ci_s - ci_{s-1}] + (\gamma^\tau - \delta^\tau)ci_0 \tag{23}$$

represents the bias of earnings for period τ. Earnings bias equals the *change* in the book-value bias in the corresponding period. While u_τ is always positive under conservative accounting, Δu_τ may be either positive or negative, depending on whether book value becomes more biased or less biased at τ relative to $\tau - 1$.

From (23), Δu_τ depends on the sequence of *consecutive changes* of past investment. As discussed above, investments in the periods immediately preceding τ have the most influence on u_τ. Thus, a key factor in determining Δu_τ is the change in investment immediately preceding date τ: $\Delta u_\tau > 0$ (i.e., x_τ understates x_τ^E) following periods of investment expansion, $\Delta u_\tau < 0$ following periods of investment decline, and $\Delta u_\tau \approx 0$ following relatively constant recent investments.

Earnings and book value combine to yield the book rate of return, $br_\tau \equiv x_\tau / B_{\tau-1}$. The book rate of return provides a biased measure of operating efficiency, or the internal rate of return $q_\tau \equiv x_\tau^E / as_{\tau-1}$. Let $d_\tau = q_\tau - br_\tau$ be the bias of the book rate of return in measuring efficiency. Then, $d_\tau \equiv [u_\tau - (1+br_\tau)u_{\tau-1}]/(B_{\tau-1}+u_{\tau-1})$.

While signing d_τ is difficult (and not necessary for the analysis below), it is clear that the sign of d_τ is related to that of $\Delta u_\tau \equiv u_\tau - u_{\tau-1}$ (as br_τ is typically much smaller than 1). Thus, in general, br_τ tends to understate q_τ following periods of investment increase, overstate q_τ following periods of investment decrease, and closely approximate q_τ following steady investments. However, when $\delta = \gamma$, we have $B_\tau = as_\tau$, $x_\tau = x_\tau^E$ and $br_\tau = q_\tau$; thus, all biases disappear.

4.2 EQUITY VALUE AS A FUNCTION OF ACCOUNTING VARIABLES

Valuation based on accounting data starts with the two basic accounting constructs, earnings and book value. Accounting data are adjusted to correct the biases introduced by conservatism; this is done by referring to past investment activities and the degree of conservatism in the accounting policy adopted. Based on information extracted from the adjusted accounting data, equity value is determined.

Repeating the same derivation process as in Section 3 yields an accounting-based valuation equation, equivalent to (16), as follows,[8]

$$V_t = \frac{1}{R-1}(x_t + \Delta u_t) + \mathcal{P}_d(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}})(B_t + u_t) + G \cdot \mathcal{C}_e(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}}), \quad (24)$$

where u_{t-1} and u_t are given by (21) and Δu_t by (23).

Equation (24) generalizes existing linear valuation models in two ways.[9] First, equity value is shown to be a non-linear function of earnings and book value, where the non-linearity arises from endogenous investment decisions. Second, (24) requires that the biases in book value and earnings be assessed separately, and these biases be used *in conjunction with* accounting data to derive information useful for valuation (and for investment decisions).

Finally, according to (24), the mapping from accounting data to equity value also relies on knowledge about parameters that characterize the firm's operating environment (the "other" information), such as growth opportunities and the frictional costs of investment (disinvestment).

5. Cross-Sectional Differences in the Properties of the Valuation Function

This section examines cross-sectional differences in the behavior of the valuation function. The analysis so far shows that value depends on anticipated future investment, which in turn depends on efficiency and growth potential. To facilitate cross-sectional comparisons, I consider three types of firms that differ in efficiency and/or growth potential:[10] low-efficiency firms, steady-state firms and growth firms. *Low-efficiency firms* face a relatively high chance of discontinuation and little chance of expansion; for these firms, the put option $\mathcal{P}_d(\cdot)$ is a significant portion of total value but $\mathcal{C}_e(\cdot)$ is negligible. *Steady-state firms* are expected to continue along the present course of operations. While efficiency is sufficiently high so that the chance of discontinuation is remote, these firms are not expected to grow either because they do not have growth potential ($G = 0$) or because their efficiency does not justify further growth. For these firms, both $\mathcal{P}_d$ and $\mathcal{C}_e$ are negligible. *Growth-firms* have the potential to grow and are efficient, so there is a high chance for realizing the growth. For these firms, $\mathcal{C}_e(\cdot)G$ is significant, but $\mathcal{P}_d$ is negligible.

5.1 THE RELATION BETWEEN EQUITY VALUE AND EARNINGS

Differentiating V_t, given by (24), with respect to x_t, with B_{t-1} held constant, and recognizing the assumption of $as_t = as_{t-1}$, we have

$$\frac{dV_t}{dx_t}\,|_{B_{t-1}} = [\frac{1}{R-1} + \mathcal{P}_d'(\cdot) + \frac{G}{B_{t-1}+u_{t-1}}\mathcal{C}_e'(\cdot)](1 + \frac{d(\Delta u_t)}{dx_t}) + \frac{\partial V_t}{\partial u_{t-1}}\frac{du_{t-1}}{dx_t}. \quad (25)$$

(25) characterizes the marginal impact of earnings on equity value, given book value.

Given book value, changes in earnings signal changes in efficiency, with possible distortions caused by measurement biases. In (25), the terms in square brackets captures the efficiency information conveyed by earnings, referred to as the *economic effect*, and the terms involving $\frac{d(\Delta u_t)}{dx_t}$ and $\frac{du_{t-1}}{dx_t}$ collectively capture the effect of accounting biases, referred to as the *measurement-bias effect*.[11] I discuss these two effects in turn.

In (25), the economic effect originates from the partial derivative $\partial V_t/\partial x_t$. If accounting measures are unbiased, the economic effect fully captures the marginal impact of earnings on value. Based on Lemmas 1 and 2 in the Appendix,

$$\frac{\partial V_t}{\partial x_t} = \frac{1}{R-1} + \mathcal{P}'_d(\cdot) + \frac{G}{B_{t-1}+u_{t-1}}\mathcal{C}'_e(\cdot) > 0. \tag{26}$$

Thus, if earnings convey efficiency information without any bias, equity value increases with earnings for any given book value, regardless of efficiency and growth potential.

To explore the second-order effect of efficiency, we have

$$\frac{\partial^2 V_t}{\partial x_t^2} = \frac{1}{B_{t-1}+u_{t-1}}\mathcal{P}''_d(\cdot) + \frac{G}{(B_{t-1}+u_{t-1})^2}\mathcal{C}''_e(\cdot) > 0, \tag{27}$$

where positivity follows from Lemma 1 in the Appendix. Thus, in the absence of accounting biases, equity value is convex in earnings, especially for low-efficiency firms and growth firms. In the former case, convexity is driven by put option $\mathcal{P}_d(\cdot)$, and in the latter case by call option $G\,\mathcal{C}_e(\cdot)$. However, for steady-state firms, both $\mathcal{P}_d$ and $G \cdot \mathcal{C}_e$ are negligible, and (27) approaches zero. In this case, value is approximately linearly related to earnings. Mathematically, this may represent a special case; but, in practice, there may be many firms of this type: operating well but lacking significant growth potential. The discussion here is summarized by Proposition 1.

PROPOSITION 1. Assume accounting measures are free of bias. Given book value, equity value increases in earnings for all types of firms, is convex in earnings for low-efficiency firms and growth firms, and is approximately linear in earnings for steady-state firms.

To determine the measurement-bias effect, one needs to examine $\frac{d\Delta u_t}{dx_t}$ and $\frac{du_{t-1}}{dx_t}$ in (25). According to (21) and (23), u_{t-1} and Δu_t are not direct functions of current earnings (x_t). This situation poses difficulties for analytically examining the behavior of $\frac{d\Delta u_t}{dx_t}$ and $\frac{du_{t-1}}{dx_t}$. In what follows, I make several assumptions about these derivatives, supported by heuristic arguments. Ultimately, the validity of these assumptions is an empirical issue.

ASSUMPTION 1. (i) $\frac{d(\Delta u_t)}{dx_t} > 0$ for firms that experienced growth in recent periods, $\frac{d(\Delta u_t)}{dx_t} < 0$ for firms that experienced downsizing in recent periods, and $\frac{d(\Delta u_t)}{dx_t} \approx 0$ for firms that experienced no recent significant changes in operating scale; (ii) $\frac{du_{t-1}}{dx_t} \approx 0$; and (iii) $\frac{d^2(\Delta u_t)}{dx_t^2} \approx 0$.

Based on Section 4.1, for firms experienced recent growth (downsizing), Δu_t tends to be positive (negative), and hence accounting earnings tend to understate (overstate) economic earnings. One may conjecture that for such firms, $\frac{d(\Delta u_t)}{dx_t}$ is positive (negative), that is, changes in economic earnings are greater (smaller) than the associated changes in accounting earnings. For steady-state firms, Δu_t is small, and hence $\frac{d(\Delta u_t)}{dx_t}$ is close to zero.[12]

Given the accounting rule, u_{t-1} is positively related to past levels of investment, as is book value B_{t-1}. Thus, u_{t-1} is likely to have a high correlation with B_{t-1}. This implies that given B_{t-1}, u_{t-1} is largely determined. On the other hand, for a given book value, differences in x_t reflect differences in operating efficiency in period t, with no obvious relation to the book value bias at t-1. This leads to the conjecture that *given book value*, the correlation between u_{t-1} and x_t is likely to be insignificant, i.e., $\frac{du_{t-1}}{dx_t} \approx 0$.

Within the theoretical setting, there is little indication of the behavior of the second-order derivative $d^2(\Delta u_t)/dx_t^2$. Given (ii) and (iii) of Assumption 1, the biases play an insignificant role in the second-order properties of the valuation function.

On the basis of (25), Proposition 1 and Assumption 1, I establish the following two predictions about the valuation impact of earnings and one hypothesis about the role of conservatism. I also discuss some related empirical evidence.

PREDICTION 1. Given book value, equity value increases in earnings for all firms.

Evidence. Burgstahler and Dichev [1997] regress V/B on x/B and find the slope coefficient to be significantly positive, except for firms in the lowest range of x/B-values (mostly firms with negative earnings, a special case discussed in the Section 6). In regressions of stock prices on earnings and book value, Collins et al. [1997] and Francis and Schipper [1999] find significantly positive coefficients on earnings.

PREDICTION 2. For a given book value, equity value is convex in earnings for low-efficiency firms and growth firms. For steady-state firms, equity value is approximately linear in earnings.

Evidence. Burgstahler and Dichev [1997] use a step-wise linear model to regress V/B on x/B, and find the average slope coefficient to be greater for firms in higher x/B-value ranges, confirming the existence of convexity within a wide range of x/B-values.[13] However, a direct test of this prediction would require an examination of valuation behavior *within* each type of firm (see the discussion in Section 7).

HYPOTHESIS 1. Given book value, conservatism increases (reduces) the marginal impact of earnings on equity value for firms having experienced recent expansions (reductions) in operating scale. For firms in a steady state of operations, conservatism has little effect on the marginal impact of earnings.

Evidence. No direct evidence is available concerning this hypothesis. Joos [1998] performs country-by-country regressions to explain stock prices by both earnings and book value, and finds that earnings coefficients are smaller in France and Germany than in the UK. However, given that earnings coefficients are also affected by operating efficiency and possibly by past investment activities, it is difficult to draw inferences from Joos's result regarding either the relative conservatism of the above countries or the impact of conservatism on the valuation equation.

Basu [1997] considers a somewhat different, but related, notion of conservatism, that earnings are more timely in recognizing bad news than good news. His result that the announcement-period abnormal returns respond more strongly to earnings increases (good news) than to earnings decreases (bad news) is broadly consistent with Hypothesis 1, which implies that the valuation impact of earnings is magnified in situations where earnings understate true economic earnings.

Adopting Basu's notion of conservatism, Ball et al. [1998] argue and provide evidence that conservatism is a universal property of accounting earnings, and that earnings in common-law countries are more conservative (i.e., more asymmetry of timeliness in reflecting bad versus good news) than in code-law countries. Pope and Walker [1999] point out that when making inferences regarding the relative conservatism of different GAAP regimes based on reported earnings, one needs to recognize possible cross-country differences in the way earnings components (such as extraordinary items) are classified.

5.2 THE RELATION BETWEEN EQUITY VALUE AND BOOK VALUE

Holding x_t constant and differentiating V_t with respect to B_{t-1},

$$\frac{dV_t}{dB_{t-1}}\mid_{x_t}= [-\mathcal{P}'_d(\cdot)\frac{x_t+\Delta u_t}{B_{t-1}+u_{t-1}}+\mathcal{P}_d(\cdot)-G\mathcal{C}'_e(\cdot)\frac{x_t+\Delta u_t}{(B_{t-1}+u_{t-1})^2}]\,(1+\frac{du_{t-1}}{dB_{t-1}})+\frac{\partial V_t}{\partial(\Delta u_t)}\frac{d(\Delta u_t)}{dB_{t-1}}. \tag{28}$$

Based on the definitions of $\mathcal{P}_d(\cdot)$ and $\mathcal{C}_e(\cdot)$, (28) can be written as

$$\begin{aligned}\frac{dV_t}{dB_{t-1}}\mid_{x_t} \quad = \quad & [\frac{q_t\cdot Prob(\nu_{t+1}\leq q_d^*-q_t)}{R(R-1)}+\mathcal{P}_d(q_t)-\frac{q_t\cdot G\cdot Prob(\nu_{t+1}\geq q_e^*-q_t)}{R(R-1)(B_{t-1}+u_{t-1})}](1+\frac{du_{t-1}}{dB_{t-1}}) \\ & +\frac{\partial V_t}{\partial(\Delta u_t)}\frac{d(\Delta u_t)}{dB_{t-1}}. \end{aligned} \tag{29}$$

Book value measures the scale of operations (asset stock) with a bias. (29) reveals how changes in book value affect equity value, holding earnings constant. The expression in the square brackets captures the valuation effect of changes in operating scale conveyed by book value, the *economic effect*, and terms involving $\frac{du_{t-1}}{dB_{t-1}}$ and $\frac{d(\Delta u_t)}{dB_{t-1}}$ collectively capture the effect of changes in accounting biases associated with changes in book value, the *measurement-bias effect*.

In (29), the economic effect originates from $\partial V_t/\partial B_{t-1}$. If accounting measures are unbiased, this effect fully captures the valuation impact of book value. From (29), this effect depends critically on efficiency (q_t). When efficiency is low, the chance of discontinuation, $Prob(\nu_{t+1} \leq q_d^* - q_t)$, is high while the chance of expansion, $Prob(\nu_{t+1} \geq q_e^* - q_t)$, is low. Thus, the first two terms in square brackets dominate the third term, implying $\partial V_t/\partial B_{t-1} > 0$; i.e., equity value increases with the asset stock.

When efficiency is high, the chance of discontinuation is low, and the chance of expansion is high, conditional on $G > 0$. In this case, the first two terms in the square brackets are dominated by the third term, so $\partial V_t/\partial B_{t-1} < 0$, implying an *inverse* relation between equity value and the asset stock, given earnings. The greater the growth potential G, the stronger this inverse relation. The intuition is as follows. The value of a growth firm primarily consists of the value from continuing operations and the value of growth opportunities. Given x_t, the value of continuing operations is determined, so the result rests on growth opportunities, $G \cdot \mathcal{C}_e(q_t)$. The key determinant of $\mathcal{C}_e(q_t)$ is efficiency, q_t. Given x_t, more asset stock is associated with lower efficiency, implying a less valuable option $\mathcal{C}_e$; therefore, equity value is negatively related to book value.

For steady-state firms, $\mathcal{P}_d$ and $G \cdot \mathcal{C}_e(q_t)$ are negligible, so $\partial V_t/\partial B_{t-1}$ is close to zero, indicating that equity value has little relation to book value, given earnings.

To examine the second-order effect of operating scale, we have

$$\frac{\partial^2 V_t}{\partial B_{t-1}^2} = [\frac{\mathcal{P}_d''(q_t)(x_t + \Delta u_t)^2}{(B_{t-1} + u_{t-1})^3} + \frac{G\mathcal{C}_e''(q_t)(x_t + \Delta u_t)^2}{(B_{t-1} + u_{t-1})^4} + \frac{2G\mathcal{C}_e'(q_t)(x_t + \Delta u_t)}{(B_{t-1} + u_{t-1})^3}] > 0, \tag{30}$$

showing that equity value is in general a convex function of operating scale, given earnings. As with earnings, the convexity with respect to book value is most prominent for low-efficiency firms and growth firms. For steady-state firms, (30) approaches zero, and the convexity disappears. I summarize the discussion in Proposition 2.

Proposition 2. Assume accounting measures are free of bias. Given earnings, equity value increases with book value for low-efficiency firms, is insensitive to book value for steady-state firms, and decreases with book value for growth firms; and equity value is convex in book value for both low-efficiency firms and growth firms, but for steady-state firms, the relation is approximately linear (and flat).

To see the valuation impact of biased book value, one needs to examine $\frac{du_{t-1}}{dB_{t-1}}$ and $\frac{d(\Delta u_t)}{dB_{t-1}}$. As before, I make assumptions about these derivatives based on intuitive arguments. Again, the validity of the assumptions is an empirical issue.

Assumption 2. (i) $\frac{du_{t-1}}{dB_{t-1}} > 0$; (ii) $\frac{d(\Delta u_t)}{dB_{t-1}} \approx 0$; and (iii) $\frac{d^2 u_{t-1}}{dB_{t-1}^2} \approx 0$.

Generally speaking, with conservative accounting, more investment in past periods leads to both a greater B_{t-1} and a greater u_{t-1}. As such, u_{t-1} is expected to have a positive correlation with B_{t-1}, i.e., $du_{t-1}/dB_{t-1} > 0$.

While B_{t-1} relates to the levels of past investments, Δu_t is determined mainly by recent *changes* in investment scale. The relevant empirical question is whether at an arbitrary date t, larger firms on average have a higher growth rate in the years immediately preceding t. In practice, firms can evolve in different patterns. For those with a sufficiently long history, book value is not likely to be highly correlated with recent changes in investment scale. To the extent this is true, the correlation between Δu_t and B_{t-1} is expected to be small.

There is little theoretical indication as to how $d^2 u_{t-1}/dB_{t-1}^2$ behaves. Given (ii) and (iii) of Assumption 2, accounting biases are insignificant in influencing the second-order valuation impact of earnings.

Based on (29), Proposition 2 and Assumption 2, I establish the following two predictions about the valuation impact of book value and one hypothesis about the role of conservatism. I also discuss some related empirical evidence.

PREDICTION 3. Given earnings, equity value increases with book value for low-efficiency firms, is insensitive to book value for steady-state firms, and decreases with book value for growth firms.

Evidence. Burgstahler and Dichev [1997] regress V/x on B/x in a step-wise linear model that allows coefficients to change across different ranges of B/x-values (firms with negative earnings are dropped). Empirically, growth firms are most likely contained in the group with lowest values of B/x (most efficient), and low-efficiency firms are likely to be in the highest B/x-value range, with steady-state firms in the intermediate range. Burgstahler and Dichev perform annual cross-sectional regressions for 1974-1994, and find that the coefficient on B/x tends to be significantly negative for firms with the lowest B/x-values, not significantly different from zero for the intermediate group, and significantly positive for the group with highest B/x-values. Also, the point estimate of the coefficient is greater in regions of higher B/x-values. Overall, these results are consistent with Prediction 3.

PREDICTION 4. Given earnings, equity value is convex in book value for low-efficiency firms and growth firms, but the relation is approximately linear (and flat) for steady-state firms.

Evidence. The evidence discussed for Prediction 3 above also confirms that equity value is generally convex in book value within a broad range of B/x-values. However, to test Prediction 4 directly, non-linear regression models may be needed.

HYPOTHESIS 2. Conservatism increases the valuation impact of book value.

Evidence. Joos [1998] finds that in regressions of stock prices on earnings and book value, the book value coefficients are greater for France and Germany than for the U.K. This result might be interpreted as consistent with Hypothesis 2 if, as Joos argues, accounting is more conservative in France and Germany than in the U.K. However, such an interpretation ultimately requires direct evidence on the relative conservatism of these countries *with respect to book value* (i.e., the extent to which book value understates asset size). Also, given Prediction 3 above, Joos's result may be confounded by the effect of operating efficiency on book value coefficients.

Basu [1997] also discusses the implication of his version of conservatism for the characteristics of book value, but the impact of his conservatism on the valuation properties with respect to book value has not been examined.

5.3 THE RELATIVE EXPLANATORY POWER OF EARNINGS VERSUS BOOK VALUE

This part of the analysis examines the relative power of earnings versus book value in explaining equity value for different types of firms. For low-efficiency firms, equity value is (approximately)

$$V_t = \frac{x_t + \Delta u_t}{R-1} + \mathcal{P}_d(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}})(B_t + u_t). \tag{31}$$

Applying the "put-call parity" condition,

$$V_t = \frac{1 - \gamma c_d}{R}(B_t + u_t) + \frac{1}{R}(x_t + \Delta u_t) + \mathcal{C}_c(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}})(B_t + u_t), \tag{32}$$

where $\mathcal{C}_c \equiv \frac{1}{R(R-1)} \int_{q_d^* - q_t}^{\bar{\nu}} [q_t + \tilde{\nu}_{t+1} - q_d^*] f(\tilde{\nu}_{t+1}) d\tilde{\nu}_{t+1}$. Thus, the equity value of a low-efficiency firm consists of the discontinuation value of assets, the earnings received before the point of discontinuation and the value of the call option to continue operations, $\mathcal{C}_c(\cdot)$. Given low efficiency, expected earnings received before discontinuation are insignificant, as is the continuation option. Thus, the value of a low-efficiency firm is derived mainly from the discontinuation value, the part that is closely related to book value (so long as bias u_t is small relative to B_t).

For steady-state firms, equity value is approximately

$$V_t = \frac{1}{R-1}(x_t + \Delta u_t). \tag{33}$$

Note that Δu_t is small for such firms. Thus, the equity value of a steady-state firm is mainly explained by earnings, and, conditional on earnings, book value adds little incremental power.

The value of a growth firm can be approximately expressed as

$$V_t = \frac{1}{R-1}(x_t + \Delta u_t) + \mathcal{C}_e(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}}) \cdot G, \tag{34}$$

where the first term represents value absent growth potential, and the second term is the net present value of growth opportunities. While the first term is related to earnings, the second term depends on efficiency. In this case, earnings *always* play a significant role in valuation. However, the significance of book value depends on G. If $G = 0$, we have a steady-state firm, and book value has little incremental power. The discussion in this subsection leads to Predictions 5, 6 and 7 below.

PREDICTION 5. For low-efficiency firms, book value is more powerful in explaining equity value than earnings.

Evidence. Collins et al. [1999] perform annual regressions to explain stock prices of negative-earnings firms from 1975 to 1992, and find that the average adjusted R^2 of model $V_i = \alpha + \beta x_i + \epsilon_i$ is 0.07, increasing to 0.41 when book value is added as an explanatory variable, indicating that the explanatory power of book value dominates that of earnings. If negative-earnings firms are reasonably representative of low-efficiency firms, this result is consistent with Prediction 5.

PREDICTION 6. For steady-state firms, earnings are a significant explanatory variable, and book value adds little incremental explanatory power.

Evidence. While this prediction has not been tested directly on a sample of steady-state firms (as defined in this paper), results in Collins et al. [1999] indirectly support it. On their samples of positive-earnings firms (a subset of which are steady-state firms), they obtain an average adjusted R^2 of 0.55 for model $V_i = \alpha + \beta x_i + \epsilon_i$, increasing to 0.59 when book value is added as a second explanatory variable.

PREDICTION 7. For growth firms, earnings and book value together explain equity value, and the incremental usefulness of book value, given earnings, increases with growth potential.

Evidence. No evidence is available concerning this prediction.

5.4 THE INTERACTIVE EFFECTS OF EARNINGS AND BOOK VALUE

Maintaining Assumptions 1 and 2, the cross derivative of the valuation function, based on (25), is

$$\frac{d^2 V_t}{dx_t \; dB_{t-1}} = -[\frac{\mathcal{P}_d''(\cdot)(x_t + \Delta u_t)}{(B_{t-1} + u_{t-1})^2} + \frac{G\mathcal{C}_e''(\cdot)(x_t + \Delta u_t)}{(B_{t-1} + u_{t-1})^3} + \frac{G\mathcal{C}_e'(\cdot)}{(B_{t-1} + u_{t-1})^2}](1 + \frac{du_{t-1}}{dB_{t-1}}) < 0, \tag{35}$$

suggesting that the marginal valuation impact of earnings decreases with book value. We have shown that equity value is increasing and convex in efficiency (q_t). A greater book value implies lower efficiency for a given level of earnings; and because of lower efficiency, an incremental increase in earnings has a smaller effect on value. This leads to Prediction 8.

PREDICTION 8. The marginal impact of earnings decreases with book value.

Evidence. The evidence discussed for Prediction 2 also supports this prediction.

6. The Empirical Relation between Stock Prices and Negative Earnings

Empirical studies have found inverse relations between equity value and negative earnings (e.g., Jan and Ou [1995], Hayn [1995], and Collins et al. [1999]). This result appears at odds with existing valuation theories (e.g., Ohlson [1995] and Feltham and Ohlson [1995; 1996]). In this section, I use my theoretical model to explain this empirical phenomenon. Assuming negative-earnings firms have low-efficiency relative to positive-earnings firms, I address this issue within the context of low-efficiency firms.

With the assumption $as_t = as_{t-1}$, the equity value of a low-efficiency firm given by (32) can be rewritten as

$$V_t = [\frac{1 - \gamma c_d}{R} + \frac{q_t}{R} + \mathcal{C}_c(q_t)] \cdot (B_t + u_t). \tag{36}$$

Earnings are given by

$$x_t = br_t \cdot B_{t-1}. \tag{37}$$

Thus, both earnings and equity value depend on asset size (or its proxy, book value) and efficiency (or its proxy, book rate of return). Changes in either factor cause changes in both equity value and earnings, thereby inducing a price-earnings relation. The overall price-earnings relation depends on the net effect of the two. I assume a high correlation (i) between B_t and as_t, (ii) between br_t and q_t, and (iii) between B_{t-1} and B_t. The conclusion is valid to the extent these conditions hold empirically.

Based on (36) and (37), both V_t and x_t increase with q_t (or br_t) for a given asset size (or book value). Thus, changes in efficiency *alone* induce a positive relation between equity value and earnings. Based on (36), given q_t, equity value increases with asset size so long as V_t is positive. As x_t is negative, so is br_t. Then, based on (37), earnings *decrease* with book value. Thus, changes in asset size induce an *inverse* relation between equity value and earnings. With two opposing factors underlying the price-earnings relation, the overall sign of this relation depends on which factor—asset size or efficiency—dominates in contributing to cross-sectional changes in equity value. I argue that for a sample of negative-earnings firms that is broadly representative in firm size, book value tends to dominate.

Based on (36), given q_t, equity value changes in proportion to book value. Within a typical empirical sample, therefore, book value *alone* can potentially induce differences in equity value across firms that are of the same order of magnitude as size differences. However, it is very unlikely that equity value differences of such magnitudes can also be caused by variations in operating efficiency alone.

To illustrate, assume $R - 1 = 0.1$ and $c_d = 0.5$. In an optimistic scenario where a currently low-efficiency firm recovers and is expected to earn a book rate of return three times the discount rate in the long run, V_t is about $3B_t$. In a pessimistic scenario where a low-efficiency firm discontinues operations immediately, $V_t = (1-c_d)B_t = 0.5B_t$. Given these parameters, the difference in efficiency between the two (extreme) cases causes a mere six-fold difference in equity value, holding B_t constant. Even if the firm in the optimistic case earns a book rate of return 10 times the discount rate, its equity value equals $10B_t$, only 20 times that in the pessimistic case. In practice, this magnitude can easily be swamped by differences in firm size.

This argument is supported by results in Collins et al. [1999]. They find that in model $V_i = \alpha + \beta x_i + \epsilon_i$, the earnings coefficient (which captures the net effect of size and efficiency) is significantly negative. When book value is added as a second explanatory variable, the coefficient on earnings (which captures the effect of efficiency) is significantly positive, as is the coefficient on book value. The adjusted R^2 of the former model is 0.07, increasing to 0.41 for the later model, confirming the dominating power of book value in explaining cross-sectional stock price differences.

7. Implications for Empirical Research

Given the valuation effects of firm-specific differences in efficiency and growth potential, regressions based on pooled cross-sectional samples might yield unreliable inferences. One implication of my model is that separate regressions estimated for samples that are homogeneous with respect to book rate of return (a proxy for efficiency) should yield more reliable inferences than pooled regressions. In this section, I develop regression models for each firm type and I comment on models commonly used in empirical studies. The models provided below employ specific forms of simplifications and are intended to serve only as general guidance.

7.1 REGRESSION MODELS FOR LOW-EFFICIENCY FIRMS

The equity value of a low-efficiency firm given by (32) is non-linear in earnings and book value. One way to convert the function into a usable form for regression analysis is to take the Taylor series expansion at a suitable point (say, $q = R-1 \equiv r_f$, for illustration). For simplicity, omit the third and higher order terms. Then,

$$
\begin{aligned}
V_t &= \frac{1}{R}(x_t + \Delta u_t) + \frac{(1-\gamma c_d)}{R}(B_t + u_t) + \mathcal{C}_c(r_f)(B_t + u_t) \\
&\quad + \mathcal{C}_c'(r_f)(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}} - r_f)(B_t + u_t) \\
&\quad + \frac{\mathcal{C}_c''(r_f)}{2}(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}} - r_f)^2(B_t + u_t). \qquad (38)
\end{aligned}
$$

Based on (38), a plausible regression model for low-efficiency firms is[14]

$$V_i = \alpha_1 + \beta_1 B_i + \gamma_1 x_i + \delta_1(\frac{x_i^2}{B_i}) + \epsilon_i, \qquad (39)$$

where $\alpha_1 = [\frac{1}{1+r_f} + \mathcal{C}_c'(r_f) - \mathcal{C}_c''(r_f)r_f](\Delta u)$, $\beta_1 = [\frac{1-c_d}{1+r_f} + \mathcal{C}_c(r_f) - \mathcal{C}_c'(r_f)r_f + \mathcal{C}_c''(r_f)r_f^2/2](1 + u/B)$, $\gamma_1 = [\frac{1}{1+r_f} + \mathcal{C}_c'(r_f) - \mathcal{C}_c''(r_f)r_f]$, $\delta_1 = \mathcal{C}_c''(r_f)/2$, and subscript i identifies firm i.

In (39), u/B is assumed to be stable across firms. Given $u/B > 0$, conservatism is shown to increase the coefficient of book value, β_1. Coefficient α_1 captures the valuation effect of earnings bias (Δu). From Section 4.1, the sign and magnitude of Δu for a specific firm depend on its past investment policies. Thus, sample firms might be partitioned on some measure of investment policy to examine whether and how the valuation impact of Δu varies with past investment activity.

If sample firms have similar values of x/B, (39) may be further simplified as

$$V_i = \alpha_1 + \beta_1 B_i + {\gamma_1}' x_i + \epsilon_i, \qquad (40)$$

where ${\gamma_1}' = \gamma_1 + \delta_1(x/B)$. (40) expresses equity value as a linear function of book value and earnings; it is a valid regression model for sample firms with approximately the same operating efficiency.

7.2 REGRESSION MODELS FOR STEADY-STATE FIRMS

The valuation equation for steady-state firms, given by (33), is easily transformed into the following regression model,

$$V_i = \alpha_2 + \gamma_2 x_i + \epsilon_i, \tag{41}$$

where $\alpha_2 = \Delta u / r_f$ and $\gamma_2 = 1/r_f$. Again, one may examine the valuation impact of Δu by partitioning the sample based on past investment activities. (41) has two general properties. First, book value has little *incremental* explanatory power over earnings. Second, cross-sectional pooling is generally valid for firms that differ in size and efficiency so long as they are all in a steady state of operations.

7.3. REGRESSION MODELS FOR GROWTH FIRMS

Applying the Taylor series expansion to (34), we get

$$V_t = \frac{1}{r_f}(x_t + \Delta u_t) + \mathcal{C}_e(r_f)G + \mathcal{C}_e'(r_f)\left(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}} - r_f\right)G + \frac{\mathcal{C}_e''(r_f)}{2}\left(\frac{x_t + \Delta u_t}{B_{t-1} + u_{t-1}} - r_f\right)^2 G. \tag{42}$$

A plausible regression model based on (42) is

$$V_i = \alpha_3 + \beta_3 x_i + \gamma_3\left(\frac{x_i}{B_i}\right) + \delta_3\left(\frac{x_i}{B_i}\right)^2 + \epsilon_i, \tag{43}$$

where $\alpha_3 = G[\mathcal{C}_e(r_f) - r_f\mathcal{C}_e'(r_f) + \frac{1}{2}\mathcal{C}_e''(r_f)r_f^2] + \Delta u/r_f$, $\beta_3 = \frac{1}{r_f}$, $\gamma_3 = G(\mathcal{C}_e'(r_f) - \mathcal{C}_e''(r_f)$, and $\delta_3 = \frac{G}{2}\mathcal{C}_e''(r_f)$. The valuation effect of earnings bias (Δu) is captured by coefficient α_3. This model cannot be reduced to a linear form similar to (40) or (41) unless sample firms have similar values of x/B and of G. Since the coefficients in (43) depend on G, estimation based on a pooled sample of growth firms would provide insight into only the *average* valuation properties of growth firms.

Models similar to (40) and (41) are sometimes used in empirical research. The discussion above suggests that (41) is appropriate for firms whose operations are expected to remain in a steady state, and (40) may be applied to firms with similar operating efficiency and, in the case of growth firms, similar growth potential. In general, non-linear models, such as (39) and (43), may be used for pooled samples that contain firms with different efficiency and growth potential.

8. Summary and Conclusions

This paper extends Ohlson [1995] and Feltham and Ohlson [1995; 1996] to include endogenous investment decisions. The model recognizes that accounting data contain information useful for guiding investment decisions, and that investment underlies value creation. In this setting, equity value equals the value from continuing the present course of operations plus the value of the option to expand or contract the scale of operations. Earnings and book value are key accounting variables for value determination, as in previous studies, but equity value with endogenous investment is shown to be non-linear in earnings and book value.

The model predicts that equity value increases with earnings for any given book value. On the other hand, given earnings, equity value is expected to increase with book value for low-efficiency firms, be insensitive to book value for steady-sate firms, and decrease with book value for growth firms. The valuation function is predicted to be convex in earnings and book value because of the option to expand or contract the operating scale. For firms expected to remain in a steady state, equity value reduces to a linear function of earnings.

The analysis shows that the relative importance of earnings versus book value in explaining equity value varies cross-sectionally with operating efficiency and growth potential. For low-efficiency firms, book value is predicted to dominate earnings while for steady-state firms, earnings are predicted to dominate book value. For growth firms, earnings and book value together explain equity value, and the usefulness of book value increases with the magnitude of the growth potential. Finally, the model offers a theoretical explanation for the negative associations demonstrated between equity value and negative earnings.

The model conjectures that accounting conservatism increases the valuation impact of book value, but may either increase or decrease the valuation impact of earnings, depending mainly on changes of investment scale in recent periods.

While much of the existing empirical evidence on valuation is generally consistent with the model's predictions, a thorough test of the model would require additional empirical analysis. For example, the model suggests that for valuation purposes, firms should be divided into different groups based on operating efficiency and growth potential, and each group should be fitted with a different regression model. Plausible regression equations are suggested for different types of firms, and conditions are identified under which linear regression equations, with earnings and book value as independent variables, may be applied. The model can also potentially be applied in a number of research areas such as earnings response studies, studies addressing the changing role of accounting data for valuation, and research investigating the impact of accounting practice on valuation.

APPENDIX

LEMMA 1. $\mathcal{P}_d'(q_t) < 0$, $\mathcal{P}_d''(q_t) > 0$, $\mathcal{C}_e'(q_t) > 0$, and $\mathcal{C}_e''(q_t) > 0$.

Proof.

By definition,

$$\mathcal{P}_d(q_t) = \frac{1}{R(R-1)} \int_{\underline{\nu}}^{q_d^* - q_t} [q_d^* - q_t - \tilde{\nu}_{t+1}] f(\tilde{\nu}_{t+1}) \; d\tilde{\nu}_{t+1}.$$

Then,

$$\mathcal{P}_d'(q_t) = -\frac{1}{R(R-1)} \text{Prob}(\tilde{\nu}_{t+1} \leq q_d^* - q_t) < 0,$$

and

$$\mathcal{P}_d''(q_t) = \frac{1}{R(R-1)} f(q_d^* - q_t) > 0.$$

Similarly, based on the definition of $\mathcal{C}_e(q_t)$, we have

$$\mathcal{C}_e'(q_t) = \frac{1}{R(R-1)} \text{Prob}(\tilde{\nu}_{t+1} \geq br_e^* - q_t) = \frac{1}{R(R-1)} [1 - \text{Prob}(\tilde{\nu}_{t+1} \leq q_e^* - q_t)],$$

and

$$\mathcal{C}_e''(q_t) = \frac{1}{R(R-1)} f(q_e^* - q_t) > 0.$$

LEMMA 2. $\frac{1}{R-1} + \mathcal{P}_d'(q_t) > 0$.

Proof.

Based on Lemma 1 above,

$$\frac{1}{R-1} + \mathcal{P}_d'(q_t) = \frac{1}{R-1} - \frac{1}{R(R-1)} \text{Prob}(\tilde{\nu}_{t+1} \leq q_d^* - q_t) > 0.$$

FOOTNOTES

* This paper has benefitted from the comments of an anonymous referee, Gary Biddle, Kevin Chen, Jevons Lee, Kay Stice and participants of accounting workshops at the Chinese University of Hong Kong and the Hong Kong University of Science and Technology. All errors are my own.

1. The point that equity value reflects a liquidation option is also found in, e.g., Berger et al. [1996], Hayn [1995] and Subramanyam and Wild [1996].

2. The link between current and future efficiency establishes the predictive usefulness of current efficiency. Assuming a random walk process simplifies the analysis; in principle the analysis can be extended to more general situations where current and future efficiency are correlated.

3. Parameter γ is equivalent to the persistence of cash flows in Feltham and Ohlson [1996]. Equations (1), (2) and (3) imply the cash flow dynamic $\tilde{cr}_{\tau+1} = \gamma cr_\tau + \kappa_\tau ci_\tau + \tilde{\nu}_{\tau+1} as_\tau$. This dynamic follows Feltham and Ohlson [1996] except that here the marginal impact on cash receipts of cash investment, and of assets in place, changes stochastically over time.

4. Assuming $as_t = as_{t-1}$ simplifies the derivation but is inconsequential to the results. As will be seen later, the non-linearity of the valuation function is driven by *anticipated* investment decisions.

5. While I assume G to be constant, the qualitative results will be re-enforced if G is an increasing function of efficiency.

6. This characterization of equity value can be generalized. Adding other

possibilities of investment/divestment at date t+1 or beyond creates more terms similar to the put or call options in (16), reinforcing the qualitative properties of the valuation function.

7. This point can be confirmed by simple numerical simulations.

8. As in Ohlson [1995] and Feltham and Ohlson [1995; 1996; 1999], all variables and parameters required for valuation are assumed to be known.

9. Equation (24) reduces to a linear valuation function equivalent to that in Feltham and Ohlson [1996] if we assume investment to follow a pre-determined linear process (without discontinuation) and if we leave cash investment at t+1 unspecified and express it simply as another variable ci_{t+1}.

10. These firm-types represent three clear-cut cases and are chosen to highlight differences across firms. The analysis, however, applies to firms with any characteristics. Moving from one firm type to another, valuation properties change gradually.

11. Empirically, $\frac{d(\Delta u_t)}{dx_t}$ and $\frac{du_{t-1}}{dx_t}$ should be interpreted as correlations of Δu_t and u_{t-1} with x_t, given B_{t-1}. Later in the analysis, $\frac{d(\Delta u_t)}{dB_{t-1}}$ and $\frac{du_{t-1}}{dB_{t-1}}$ should be similarly interpreted as correlations of Δu_t and u_{t-1} with B_{t-1}, given x_t.

12. Empirically, it is implausible to have $\frac{d(\Delta u_t)}{dx_t} < -1$ or, equivalently, $\frac{dx_t^E}{dx_t} < 0$, since this would imply that an increase in accounting earnings generally signals a *reduction* in economic earnings.

13. Freeman and Tse [1992] also find evidence of non-linearity but in the context of market reactions to unexpected earnings; the relation they consider is

about changes in, rather than levels of, the related variables.

14. For simplicity, both the bias in book rate of return and the distinction between the beginning and ending book value are ignored in arriving at (39).

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