# Diversification and the Accounting for New Projects On or Off the Balance Sheet

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**Abstract**

A debt financing transaction that is structured to avoid explicit liability recognition is known as off-balance sheet financing (OBSF). Purported benefits from OBSF include raising cheaper debt by guaranteeing debt repayments unencumbered by current debt contracts, maintaining desired debt-to-capitalization ratios, preserving credit ratings and future borrowing capacity, funding projects beyond approved capital budgets. Despite its appeal, many firms choose conventional financing (balance sheet financing or BSF). We demonstrate that allowing firms the choice between OBSF and BSF can play a positive informational role, notwithstanding the argument that permitting this choice compromises the representational faithfulness of the balance sheet. In the context of raising funds for a new project, we derive an equilibrium in which the firms divide themselves between OBSF and BSF in a manner that provides useful information for valuing the firms' stock prices. Consistent with anecdotal evidence we demonstrate riskier firms use OBSF and the projects financed are the riskier projects.

**Diversification and the Accounting for New Projects On or Off the Balance Sheet**

# 1 Introduction

A debt financing transaction that is structured to avoid explicit liability recognition is known as off-balance sheet financing (OBSF). While GAAP limits its use, there are numerous purported benefits to keeping debt off the balance sheet, motivating some firms to engage in OBSF. Benefits include raising cheaper debt by guaranteeing debt repayments unencumbered by current debt contracts, maintaining desired debt-to-capitalization ratios, preserving credit ratings and future borrowing capacity, funding projects beyond approved capital budgets.[1]

Despite the appeal of OBSF, many firms choose conventional financing (balance sheet financing or BSF). While the inability to satisfy GAAP requirements for OBSF may explain why some of these firms choose conventional financing, we demonstrate informational considerations may also be a factor. In particular we show that allowing firms the choice between OBSF and BSF can play a positive informational role, notwithstanding the argument that permitting this choice compromises the representational faithfulness of the balance sheet. In the context of raising funds for a new project, we derive an equilibrium in which the firms divide themselves between OBSF and BSF in a manner that provides useful information for valuing the firms' stock prices. Consistent with anecdotal evidence we demonstrate riskier firms use OBSF and the projects financed are the riskier projects.[2]

The recognition versus disclosure debate has a long history in the accounting literature and is an important concern in the setting of standards. By definition, OBSF activities are disclosed in the footnotes to the financial statements unlike BSF which is accompanied by accounting recognition. This paper argues that the choice between disclosure and recognition can itself play an important informational role.

In our model, we examine a firm with debt on its balance sheet that has a new project opportunity. Given that the new project is to be financed with debt, we ask whether the firm would prefer project financing, resulting in off-the-balance sheet presentation of the debt, or subordinate debt and presentation on-the balance sheet. Closely related to this accounting/financing choice is the firm's project choice; does the firm prefer a project whose

[1] See, for instance, Toll [1996]. This article provides many recent examples of off-balance sheet financing involving Bank of America, EnCap Investment.

[2] See Nevitt [1979] for a discussion of the evidence.

cash flows are closely related to the cash flows from existing operations, or would the firm prefer to diversify.

When a firm has debt in its capital structure, clauses in the debt contract affect the cost of new debt and may influence the nature of new investments undertaken. This debt overhang problem is well recognized in the finance literature (Myers [1977]). The firm can avoid this debt overhang problem through project financing; the seniority clauses are circumvented by setting up the project as a separate legal entity and listing the new debt off the parent's balance sheet. Thus, in our model the benefit of OBSF is its usefulness in solving the debt overhang problem.[3]

To capture the informational role of the two accounting treatments, OBSF and BSF, we examine a setting in which shareholders have less information about the firm than the firm's manager, but institutional lenders are better informed than shareholders (Rajan & Winton [1995]). Institutional lenders, prior to setting the terms of a loan, can and do demand credit information that is not publicly available. Thus the type of financing and the terms of a loan granted by the better informed institutional lender can provide information about the firm, information which will be reflected in the firm's stock price.

To see how this valuation effect may impact a firm's financing and investment choices, consider a firm which, prior to taking the project, is unlikely to be bankrupt. For this firm the extra cost of using conventional debt over project financing is small. While conventional debt will enable existing debt holders to receive some of the cash flows from the project, because this firm is not very risky, the extra expected payments to existing debt holders are small. Hence, this firm may prefer conventional debt where the loan terms will convey information about its low risk to shareholders. If the firm has a choice among investments, the firm may want to select a project that diversifies away some of its risk. The advantage to diversification

[3]In this paper, the accounting treatment is linked to the financing choice. Conventional debt necessarily appears on the balance sheet. With project financing, a separate legal entity is set up. While this does not guarantee that consolidation can be avoided, as noted above, there are numerous reasons firms want to structure the transaction to allow off balance sheet presentation. In the literature on project financing, presenting the debt off the balance sheet is treated as a given objective. For example, Hoffman [1989b] lists one of the benefits of project financing as "the potential for using off-balance-sheet accounting techniques for project commitments." (p. 186). In the preface to the book Project Financing (Nevitt [1979]), it is stated: "The term 'project financing' was first used in the early editions of this book to segregate and describe certain kinds of instruments and certain types of transactions with unique characteristics which enabled promoters of a project financing transaction to shift debt burden, operating risk and accounting liabilities to third parties, while at the same time retaining some of the benefits of the project. In the ensuing years the term project financing has acquired a unique definition as a financing with off-balance sheet and shifted liability characteristics."

is not an increase in expected cash flows; the firm benefits when it gets the same loan terms as another firm which is even less risky and thereby is confounded in shareholders' eyes with this firm. Thus, for a low risk firm, a project that diversifies its risk financed with conventional debt is desirable.

In summary, when shareholders have less information about a firm than the firm's manager and the firm's banker, we demonstrate that in equilibrium:

- Less risky firms use conventional debt, with on-the-balance sheet presentation, to provide information to the stock market about the firms' low risk characteristics. More risky firms resort to project financing. The debt overhang issue is too severe for this group to want to use conventional debt.

- If small differences in diversification are not observable, some firms may prefer to use conventional debt, and to diversify, in order to get more favorable loan terms (like higher bond ratings) when the more favorable loan terms affects shareholders' perceptions of the firm's expected cash flows. Moderately risky firms who would use off-balance sheet financing if no project choice were available may switch to conventional debt if, through diversification and conventional debt, they can favorably affect their stock values.

Thus, our results suggest the project financing option, in conjunction with its accounting treatment, plays a positive informational role in equilibrium, This result is in the same spirit as Dye & Verrecchia [1995] which shows that expanding managerial reporting discretion ameliorates internal agency problems. Our paper is also similar to Levine [1996] which demonstrates that allowing differential accounting treatments for compensation instruments that have similar motivational impacts (such as stock options and stock appreciation rights) can enhance informational efficiency in equilibrium.

The model we present in this paper builds on Berkovitch & Kim [1990]. Assuming the returns on a new project are perfectly correlated with the firm's existing assets, Berkovitch and Kim examine the effects of seniority rules on a firm's decision to invest. They study a firm's incentives to over- and under-invest as a function of the seniority rule. In contrast, we are concerned with how the conventional subordinate debt versus project financing choice, and the diversification decision, is influenced by consequent valuation implication.

Our paper is related to Shah & Thakor [1987] which examines the optimal capital structure when project financing is an option. Shah and Thakor is also motivated by the stylized fact that "many investments utilizing project financing appear to be highly risky." (p. 212).

In Shah and Thakor, the asymmetry of information is between the firm's management and new creditors, while in our model, the asymmetry of information is between the firm and shareholders. Further, in Shah and Thakor, shareholders play no role, and disclosure is a non-issue. In our model, the financing choice is inextricably linked to the information the firm wishes to convey to the financial market.

Finally, the conference on Off-Balance Sheet Financing Activities, reported in a special issue of the *Journal of Accounting, Auditing & Finance* (1989), discussed many of the institutional features used in the paper. Of particular interest is James [1989] who describes the off balance sheet transactions of banks. James indicates that the characteristics of off balance sheet activities differs from activities on the balance sheet. This is similar to our observation that on- and off-balance sheet loans convey different types of information. Rajan [1992] and Rajan & Winton [1995] both model settings in which institutional lenders are informed lenders with the ability to obtain information about the project for which the debt is being raised. This informedness of the institutional lender plays a crucial role here in determining what type of information is revealed through financial statements via on and off-balance sheet debt.

The paper proceeds as follows. In Section 2 we describe the structure and the assumptions of our model. In Section 3, we analyze the base case in which there are no information asymmetries between a firm, its shareholders and its debtholders, and establish the motivation for the rest of the analysis. We then introduce asymmetric information in Section 4 and demonstrate a demand for BSF and diversification. In the concluding section, we discuss the impact of relaxing our assumptions. In appendix A, we present an overview of the project financing transaction.

# 2 Model

We construct a two period model of a firm. The firm is an on-going entity and at time zero has assets and debt in place. In the first period, the firm is faced with an investment opportunity; the firm must decide (i) whether to undertake a new project, (ii) which project to choose, if a choice is available, and (iii) what type of debt financing to employ. In the second period, returns on the existing assets and the new investment, if it was undertaken, are realized and debt payments made. To concentrate on the informational impact of the accounting treatment choice, throughout the paper we assume (1) firms are able to structure their debt transactions to satisfy GAAP requirements for OBSF, and (2) the cash flows from the investment are not affected by the structure of the debt. We model a simple choice for firms – circumventing the debt over-hang issue versus differential information revelations about their risk characteristics.

*Asset structure*

The firm's existing assets yield a return of $X_L$ with probability $p$ and $X_H$ with probability $(1-p)$, with $0 < X_L < X_H$. We refer to the probability $p$ as the firm's risk factor. Letting $\mu_X(p)$ be the expected return on the firm's existing assets, $\mu_X(p) = (1-p)X_L + pX_H$. The face value of the firm's existing debt, $F$, is sufficiently large that the firm will be bankrupt if the lower return occurs, or

**Assumption 1.** $0 < X_L < F < X_H$.

For $I$ dollars, the firm has the opportunity to invest in a new (weakly) risky asset which will return either $Y_L$ or $Y_H$ where

**Assumption 2.** $0 \leq Y_L \leq I \leq Y_H$.

We assume the firm's returns and the face value of the debt are such that:

**Assumption 3.** $F > X_L + I$.

Together assumptions 2 and 3 imply the project is sufficiently small to insure that existing debt holders are not fully repaid when the project and existing assets both earn low returns even if existing debtholders capture all of the returns from the new project. Hence, even when the seniority clause is in effect, and the firm chooses to diversify, current debtholders face some risk.

If the returns on the project and the returns on the firm's existing assets are perfectly correlated (positively or negatively), only two states occur. When the project and the existing assets are not perfectly correlated, there are four possible states for the firm – $X_H + Y_H$, $X_H + Y_L$, $X_L + Y_H$, and $X_L + Y_L$. Assumptions 1 and 2 imply, $X_H + Y_H$ is the most favorable outcome and $X_L + Y_L$ is the least favorable outcome. The following assumption yields a unique ordering of the four outcomes.

**Assumption 4.** $X_H + Y_L > X_L + Y_H$.

Assumption 4 implies the returns from the firm's existing assets determine which states are more favorable for the firm, or existing assets "dominate" the new project in determining the firm's fortunes. Investing in the new project does not alter the nature of the existing business, changing what is a good outcome for the firm.

All firms have the same cost for the new investment ($I$), the same return if the project succeeds ($Y_H$), the same face value of existing debt ($F$), and same returns on the existing project ($X_H$ and $X_L$). Where firms differ is in their risk factor ($p$) and the return on the project in the case of a poor outcome ($Y_L$). Firms may also select different degrees of diversification for their projects or, equivalently, different correlation coefficients between the returns on the project and the returns on existing assets ($r$). Thus a firm is identified by the triple $(Y_L, p; r)$. The basic thrust of our analysis is the impact of the information asymmetry about $Y_L$ and $p$ (and $r$) between the firm and its shareholders on the firm's choice of a financing method and project. A firm's financing choice, accounting treatment, and project choice could all potentially reveal information to shareholders and affect valuation.

To keep our analysis simple, we assume that projects in the firm's investment opportunity set have the same risk as its existing assets. Formally, the probability of the project's lower return $Y_L$, is also $p$; letting $\mu_Y(p)$ be the expected return on the new asset, $\mu_Y(p) = (1 - p)Y_L + pY_H$. Thus, if a firm's existing assets pose little risk, the firm is assumed to have access to low risk projects. The low risk in the existing assets may be due to superior managerial ability or organizational structure which carry over to the new project. While all available project options for a firm yield the same expected returns, these projects are characterized by different correlation coefficients ($r$) between their returns and the returns on the firm's existing assets. This feature of the model enables us to focus squarely on management's diversification/consolidation decision and the interaction between this decision and management's financing/reporting choice.[4]

Let $\delta_{ij}(p; r)$ be the probability that the outcome $X_i + Y_j$ occurs ($i, j = L, H$). Given that the expected returns from the existing assets $\mu_X(p)$ and the expected return from the new project $\mu_Y(p)$ are independent of the correlation coefficient, $r$, $\delta_{ij}(p; r)$ can be computed as

[4]There is a second reason for assuming the firm's existing assets and project options have the same risk. This paper is about asymmetric information and the choice of debt based on the information conveyed in the loan's terms. As will become clearer later in the paper, if there were no relationship between the expected returns on the project and the firm's existing assets, this would make OBSF less attractive; for informational reasons more firms would use BSF. By assuming the same risk parameter (p) for both the existing assets and all possible projects, we bias the model in favor of the firm's using OBSF. Despite this, we find less risky firms using BSF.

follows (refer to the appendix):[5]

$$
\begin{aligned}
\delta_{HH}(p;r) &= (1-p)\,[1-p(1-r)]\,, \\
\delta_{HL}(p;r) &= (1-p)\,p(1-r), \\
\delta_{LH}(p;r) &= (1-p)\,p(1-r), \\
\delta_{LL}(p;r) &= p\,(p+r(1-p))\,, \\
\text{where } r &\in [r_{\min},1], r_{\min} = Max\{-\frac{1-p}{p}, -\frac{p}{1-p}\}.
\end{aligned} \tag{1}
$$

*Financing*

We consider only two types of financing for the project. If the project is undertaken, it is funded using either project financing with off-the-balance sheet presentation or conventional debt listed on the balance sheet. Our goal is not to provide a characterization of the firm's optimal capital structure choice. We focus more narrowly on the trade-offs between project financing versus general debt in an attempt to understand some of the forces that drive the use of different accounting treatments. In the conclusion we briefly discuss the implications of allowing equity or internal financing.

In addition to the two loan types being booked differently, the different financing methods have different cash flow implications for the firm:

1. Conventional debt, listed on-the-balance sheet (labeled hereafter as BSF), is assumed to be strictly subordinate to existing debt.
2. Project financing, listed off-the-balance sheet (labeled hereafter as OBSF), gives lenders who finance the project first claim to the assets of the new project in the event of default, but no claim to the firm's existing assets.

The face value of the debt for the two types of financing depends on the above cash flows, the nature of the debt market, and the states in which the firm is bankrupt when the project is taken. We assume that debt financing comes from risk-neutral institutional lenders who operate in a competitive market. With no loss in generality, we set the risk free rate of return

[5] Because all projects promise the same expected return $\mu_Y(p)$, independent of $r$, not all correlation coefficients in the interval $[-1,1]$ are feasible for a firm with a given $p$. For example, suppose $p=0$; since both $X_H$ and $Y_H$ occur with probability 1, it is impossible to generate a project which is negatively correlated with existing assets. Restricting the probabilities $\delta_{ij}(p;r)$ to being non-negative, one can compute a minimum feasible correlation coefficient $r_{\min}$for each $p$.

to be zero. We also assume the project reduces the probability of the firm being bankrupt if BSF is used. Without the project the firm is bankrupt when the cash flows equal $X_L$. With the project the firm is bankrupt only in state $X_L + Y_L$. Thus, if the project is selected there are some advantages to diversification; the firm will be solvent more frequently.

Let the face value of the conventional debt or BSF for firm $(Y_L, p; r)$ be denoted by $B(Y_L, p; r)$. If the firm were to be solvent in all states except the state $X_L + Y_L$, the face value $B(Y_L, p; r)$ can be computed as

$$\begin{aligned} I &= (1 - \delta_{LL}(p; r))B(Y_L, p; r), \text{ or} \\ B(Y_L, p; r) &= \frac{I}{(1 - \delta_{LL}(p; r))}. \end{aligned} \tag{2}$$

Given Assumption 4, the firm will be solvent in all states but $X_L + Y_L$ if $X_L + Y_H \geq F + B(Y_L, p; r)$ for all $Y_L, p$ and $r \in [r_{\min}, 1]$, or if $p \leq \overline{p}(r)$, where[6]

$$\begin{aligned} \overline{p}(r) &= \begin{cases} \frac{[-r(X_L+Y_H-F)+\Omega]}{2(1-r)(X_L+Y_H-F)} & \text{for } r \in [r_{\min}, 1) \\ \frac{X_L+Y_H-F-I}{X_L+Y_H-F} & \text{for } r = 1, \text{ where} \end{cases} \\ \Omega &= \sqrt{4(1-r)(X_L+Y_H-F)(X_L+Y_H-F-I) + r^2(X_L+Y_H-F)^2}. \end{aligned}$$

Thus, for the firm to be solvent in all states except the state $X_L + Y_L$ with BSF, we need the following assumption.

**Assumption 5.** $p \leq \overline{p}(r)$, for all $r \in [r_{\min}, 1]$.

Assumption 5 implies a lender is always willing to give the firm BSF; the firm is solvent in some states, and hence the lender's terms can always be met.

Off balance sheet financing allows the manager to separate the returns on the project from the returns on the firm's existing assets. The firm is able to escape the seniority clause of existing debt, but the new lenders have no recourse to returns on existing assets. As the cash flows are separate, the correlation between the returns of the project and existing assets is irrelevant to the financing costs. Let $O(Y_L, p)$ denote the face value of the off balance sheet

[6]Because of this upper bound on $p$, the project is always socially desirable and is undertaken. Hence, questions addressed in the finance literature about over- and under-investment are moot in this paper. A paper that looks at such issues in a similar setting is Berkovitch and Kim (1990).

debt; $O(Y_L, p)$ is computed as follows:

$$I = (1-p)O(Y_L,p) + pY_L, \text{ or}$$
$$O(Y_L,p) = \frac{I}{(1-p)} - \frac{pY_L}{(1-p)}. \tag{3}$$

Assumption 5 also implies a lender is always willing to grant the firm OBSF. Straightforward calculations indicate that $p \leq \overline{p}(r)$ for all $r \in [r_{\min}, 1]$ insures that $O(Y_L, p) < Y_H$ (see appendix for calculations), or the firm's cash flows in the high state are sufficient to guarantee the loan. Thus, in this paper, both types of financing are forthcoming. There is no issue of a firm selecting one type of financing because the other is not available.

In summary, the sequence of events is as follows:

1. The firm (i.e., the manager) chooses among its three strategies: (i) Take the project and use BSF, (ii) take the project and use OBSF, and (iii) do not undertake the project.

2. The firm issues its financial report in which, if a new loan was acquired, the loan is presented either on or off the balance sheet.

3. The stock price for the firm's shares is determined.

4. In the future the firm's and project's returns are realized, and payments made to debtholders.

The manager acts to maximize the first period or current share price of the firm. In a world of complete information this is equivalent to maximizing the firm's cash flows.

# 3 Analysis

In the following we determine the preferred financing and diversifying options for each firm type. We begin by assuming perfect symmetric information among all parties. This provides a benchmark and confirms the intuition provided in the introduction: in a world of perfect information, a firm using BSF strictly prefers to not diversify, and OBSF is preferred to BSF for any level of diversification. In the subsequent two sections we introduce asymmetric information, assuming shareholders are not perfectly informed about the parameters of the firm or its new project.

### 3.1 Symmetric (perfect) information

With perfect information, and Assumption 5, insuring the firm is bankrupt only when the state $X_L + Y_L$ occurs, the expected profit of a firm $(Y_L, p; r)$ with BSF is

$$\begin{aligned}\pi_S^B(Y_L, p; r) \quad = \quad & \delta_{HH}(p; r)\left[X_H + Y_H - F - B(Y_L, p; r)\right] + \\ & \delta_{HL}(p; r)\left[X_H + Y_L - F - B(Y_L, p; r)\right] + \\ & \delta_{LH}(p; r)\left[X_L + Y_H - F - B(Y_L, p; r)\right], \end{aligned} \tag{4}$$

where the superscript $B$ denotes BSF, and the subscript $S$ represents symmetric information. The expected profit of the firm with off balance sheet financing is

$$\pi_S^O(Y_L, p) = (1 - p)\left[X_H + Y_H - F - O(Y_L, p)\right]. \tag{5}$$

where the superscript O denotes OBSF.

In the absence of any informational asymmetries, there is no information to be conveyed through the firm's financing decision. The firm's project and financing choices depend solely on their impact on the firm's stock price (or expected cash flows with symmetric information). First consider the effect of diversification on the financing costs. As noted above, diversification has no impact on OBSF costs. But diversification does reduce BSF costs. The probability $\delta_{LL}(p; r)$ is increasing in $r$, and hence $B(Y_L, p; r)$ is increasing in $r$.

**Observation 1.** *Given Assumptions 1-5, diversification reduces the probability of bankruptcy and the cost of balance sheet financing (BSF).*

Is one form of financing always the cheapest? In general one cannot say, but one can readily show:

**Observation 2.** *Given Assumptions 1-5, OBSF is (weakly) cheaper than BSF for $r = 1$, while BSF is (weakly) cheaper than OBSF when $r = r_{\min}$* and $0 \leq p \leq \frac{I}{2Y_L}$.

When $r = 1$, there are only two possible outcomes for the firm - $X_H + Y_H$ occurs with probability $1 - p$, and $X_L + Y_L$ occurs with probability $p$ . When BSF is used and $X_L + Y_L$ is realized, given Assumption 3, all the proceeds from the new debt $(Y_L)$ go to the existing debtholders; lenders financing the new project receive nothing. However, with OBSF, the existing debtholders have no claim on returns from the new project, and lenders financing the new project are repaid $Y_L$. In a competitive debt market, this feature of OBSF translates into cheaper debt. When $r = r_{\min}$, diversification sufficiently reduces the cost of BSF financing that it becomes the cheaper financing option as long as the firm is not too risky.

Yet despite the cheaper financing costs with diversification, and the reduced probability of the firm being bankrupt, diversification is not desirable, and firms do not want to use BSF.

**Proposition 1** *With symmetric (perfect) information, if a firm uses BSF, it strictly prefers to not diversify ($r = 1$). If a firm uses OBSF it is indifferent to the extent of diversification, $r$. Firms strictly prefer OBSF to BSF for any $r$.*

Why is diversification not desirable with BSF? As noted above, there are two positive effects to diversification–cheaper financing costs and the firm is solvent more frequently. But there is a third effect: current debt-holders are repaid more frequently. The total expected returns for the firm $\mu_X(p) + \mu_Y(p)$ are the same regardless which project is chosen, but diversification affects the division of this constant amount among current debt-holders, new lenders and shareholders. New lenders receive a constant expected payment since they operate in a competitive debt market. Since diversification means the firm is bankrupt less frequently, current debt-holders are repaid more frequently. Hence current debt-holders receive an extra expected payment which reduces the shareholders' portion.

### 3.2 Asymmetric information - $r$ observable

In this section we introduce an informational asymmetry between the firm and its shareholders. Specifically, while the values of $X_L$, $X_H$, $F$, $I$, and $r$ are public information, we assume shareholders cannot observe the values of $p$ and $Y_L$.[7] In the next section we add to the asymmetric information by restricting the observability of $r$. Shareholders' beliefs regarding the unobservable $p$ are represented by a uniform distribution $p \sim U[0, \overline{p}]$. Shareholders' beliefs regarding $Y_L$ are represented by the uniform distribution $Y_L \sim U[0, I]$.

What information do creditors have? Institutional lenders, before setting the terms of a loan, can and do demand information beyond that provided by the firm's financial statements. To operationalize this observation simply, we assume the lender can learn perfectly the parameters $p$ and $Y_L$. This information is available only to the lender who is approached for the loan, and the lender is assumed to be legally bound to maintain confidentiality. [8] These

[7] To maintain tractability we assume only two dimensions to the unobservable information about the firm. With a small number of unobservable characteristics one might wonder why the firm does not voluntarily disclose information it wishes released. However, given the numerous firm characteristics which a banker must actually process prior to setting the interest rate, plus the potential proprietary nature of some of the information, we ignore the disclosure option.

[8] As insider trading is illegal, we do not allow individuals within the lending institutions to trade on their own or on the lending institution's account. It is well known that it is illegal for lending institutions to trade

assumptions are consistent with [Rajan & Winton, 1995], who model institutional lenders as being in a better position to monitor a firm than other stakeholders. We also assume the creditor's informational advantage does not give him any monopoly power in pricing the loan. In the competitive debt market, the mere possibility that the firm may approach another lender for the loan is enough to ensure that any lender will price the loan rationally and competitively. Thus, the pricing equations for BSF and OBSF debt are still given by equations 2 and 3, respectively.

The informational asymmetries between the firm and its shareholders may provide firms a reason to use BSF. The type of loan and the terms of the loan are available in the firm's financial statements and provide information to the stock market about the values of $p$ and $Y_L$. Thus, in choosing between OBSF and BSF, management weighs the cost of financing the loan against the valuation implication for the shareholders' expectations of the firm's share price given the information revealed by the form and the face value of the loan.

Proposition 1 showed that absent information asymmetries, it is in the firm's best interest not to diversify when using BSF. The following observation addresses the choice of $r$ when $p$ and $Y_L$ are not observable to shareholders.

**Observation 3.** *Given assumptions 1-5, if the extent of diversification, $r$ , is observable to shareholders, the firm will choose to not diversify ($r = 1$) even if the parameters $p$ and $Y_L$ are unobservable to shareholders.*

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on nonpublic information gained from their clients:

> The U. S. Supreme Court has stated that an underwriter, acountant, lawyer, or consultant engaged by an issuer takes on the role of 'temporary insider' if the issuer expects the outsider to keep undisclosed information confidential and if the reltionship with the issuer is of such a nature that it implies confidentiality. For the most part, case law has established that individuals that have a *fiduciary* relationship with an issuer clearly must adhere to the disclose-or-abstain rule. (from SEC Regulation of Public Companies, Allan B. Afterman, Prentice Hall, New Jersey, l995, p. 81).

To avoid prosecution, lending instituions with both investing and lending departments must make special arrangements to prohibit the lending department's information from being used by the investing areas:

> [i]nvestment advisers, broker-dealers, and banks have developed policies against insider trading and procedures for policing those policies, including so-called Chinese walls where necessary to permit multi-purpose fuirms to continue their businesses by sealing off the flow of nonpoublic information from one department to another. (Modern Investment Management and the Prudent Man Rule, Bevis Longstretch, Oxford University Press, New York or Oxford, l986, p. 71.)

When $r$ is observable, shareholders can infer the value of $p$ from the reported face value of the BSF debt, $B$. Since the face value of the BSF debt, $B$, does not depend upon $Y_L$, the pricing equation (2) is invertible. The ability to infer the value of $p$, together with the intuition from Proposition 1, establishes the observation. Given Observation 3, we set $r = 1$ for the rest of this section. Hence, while this section can provide a rationale for firms using BSF, it does not address diversification choices.

The extent to which the reported balance sheet debt conveys information about the parameters $p$ and $Y_L$ can be determined only in the context of an equilibrium. In addition to knowing which other firms could have the same loan terms, it is important to know in equilibrium which other firms choose that loan form. Thus shareholders make inferences regarding a firm's type, $\{Y_L, p\}$, from two pieces of information (i) the face value of the debt (i.e., the magnitude of $B$ or $O$), (ii) the equilibrium sets of firms with the same loan type and face value, and those not undertaking the project. Shareholders' valuation of the firm's current share price, based on this information, can be expressed as

$$\begin{aligned}
\pi^B &= (1 - E[p|B])[X_H + Y_H - F - B], \\
\pi^O &= (1 - E[p|O])[X_H + Y_H - F - O], \\
\pi^N &= (1 - E[p|N])[X_H - F]. \qquad (6)
\end{aligned}$$

where $N$ represents those firms who do not undertake the project.

We look for a sequential equilibrium where each firm type's choice maximizes the firm's current share price, given consistent shareholders' beliefs about the types of firms who would select each option.

If a firm takes the project, its financing choice depends on the financing costs and valuation implications of each alternative. Consider the nature of the information conveyed by each of the financing options. Given the face value of the BSF (OBSF) debt, $B(O)$, shareholders can identify the set of firms $\{Y_L, p\}$ who would get BSF (OBSF) loans with that face value. Given BSF (OBSF) where the face value of the debt is $B(O)$, the iso-face value curve, or the set of all firms $\{Y_L, p\}$ who would get BSF (OBSF) debt with face value $B(O)$, is given by:

$$\begin{aligned}
p_B(Y_L; B) &= \frac{B - I}{B}, \\
p_O(Y_L; O) &= \frac{O - I}{O - Y_L}. \qquad (7)
\end{aligned}$$

A set of iso-face value curves for BSF (OBSF) debt is illustrated in Figure 1.

(Figure 1 here)

As shown, the iso-face value curves for BSF debt are horizontal lines in the $(Y_L, p)$ space, while the iso-face value curves for OBSF debt slope upward. Given the face value $B$ for BSF debt, shareholders know the risk, $p$, of the firm. On the other hand, the face value $O$ for OBSF can correspond to a very risky firm (large $p$ and large $Y_L$) or a firm with very little risk (small $p$ and small $Y_L$).

**Observation 4** *For $r = 1$, the face value of BSF debt is independent of $Y_L$ and therefore reveals p perfectly. However, the face value of OBSF debt is influenced by both p and $Y_L$, and shareholders cannot make clear inferences about either p or $Y_L$.*

Given that BSF and OBSF convey different information to shareholders, affecting their valuation of the firm, it may be beneficial for a firm to choose BSF even though OBSF is cheaper (Observation 2).

**Proposition 2** *Given assumptions 1-5 and shareholders' imperfect information about p and $Y_L$, in any equilibrium, the least risky firms always select BSF and the most risky firms always select OBSF.*

Consider a firm whose risk is very small; for example, a firm with $p = \varepsilon$. (See firm X in Figure 2.) If this firm were to engage in BSF, Observation 4 tells us $p$ would be revealed to shareholders. If it were to engage in OBSF, shareholders would perceive it to be of a higher risk because the OBSF curve for that same firm is upward sloping. The consequent valuation implications more than outweigh the benefit from reduced financing cost, since the extra financing cost with BSF is very small when a firm is not very risky. Hence, it is not beneficial for the least risky firms to choose OBSF.

(Figure 2 here)

Consider next a firm whose risk is very high; for example a firm with $p = \overline{p}$. (See firm Y in Figure 2.) Such a high risk firm would not face adverse valuation consequences if it were to engage in OBSF. With BSF, the firm's true risk ($\overline{p}$) is revealed; by using OBSF the firm reduces shareholders' perceptions of its risk since all other firms on the same OBSF face-value curve must have $p$'s less than $\overline{p}$. In addition, OBSF is always cheaper; hence, both valuation and financing considerations weigh in favor of OBSF for a high risk firm.

While Proposition 2 characterizes the behavior of high and low risk firms in any equilibrium, it does not establish the existence of an equilibrium. Proposition 3 addresses this issue.

**Proposition 3** *There exists an equilibrium in which the riskier firms select OBSF and the less risky firms select BSF.*

In the proof of Proposition 3, we derive a boundary that splits the set of firms in the $(Y_L, p)$ space into two regions as illustrated in Figure 3.

Off-balance sheet financing is preferred in this equilibrium by riskier firms above and to the right of this boundary (the region **OBSF** in the figure), and balance sheet financing is preferred by less risky firm below and to the left of this boundary (the region **BSF** in the figure). While we have not established the uniqueness of this equilibrium, the existence of *an* equilibrium, together with the result that a non-empty subset of firms will choose each forms of financing in *any* equilibrium (Proposition 2), provides one rationale for these two forms of financing co-existing in practice. The proposition is also consistent with the stylized fact that riskier firms use OBSF.

### 3.3 Asymmetric information - $r$ imperfectly observable

In this section we relax the assumption that the diversification choice $r$ is observable to shareholders. Lenders, as in the last section, remain fully informed. By limiting shareholders' ability to observe a firm's diversification choice, we show there can be a demand for diversification. Despite its cost, a firm may choose to reduce its financing charges through diversification, and thereby influence shareholders' perceptions of the firm's risk, increasing the value of its stock.

It would be unrealistic to assume that $r$ is totally unobservable because outsiders can form some assessment of the extent of the synergy (or lack thereof) between a new project and existing assets upon observing the nature of the new project. For instance, outsiders would rationally assess $r$ to be higher for an automobile manufacturing company acquiring one of its suppliers than for an auto manufacturer acquiring a grocery store. On the other hand, shareholders probably could not determine the exact correlation between the auto manufacturer's returns and the supplier's returns. To implement imperfect observability of $r$, we make the following assumption.

**Assumption 6.** Two firms $i$ and $j$ cannot be distinguished as having different correlation coefficients for their new projects if $|r_i - r_j| < k$, for a fixed $k > 0$.

The lack of perfect observability of $r$ provides a firm with an additional degree of freedom in influencing the trade-off between valuation implications and financing costs associated with its financing choice. In the previous section we established if $Y_L$ and $p$ are not observable, less risky firms would select $r = 1$ and use BSF to provide financial information that they are not very risky firms. If, in addition, there is limited observability of $r$, a riskier firm may benefit from a valuation perspective by opting for BSF and choosing $r < 1$ if this enables the firm to pool with other less risky firms choosing BSF. Demonstrating this intuition with a continuum of firm types has proven intractable; hence, we use an analytically more manageable three firm model.

Consider three firms $(Y_L^i, p_i), i = 1, 2, 3$, that lie on the same OBSF iso-face value curve with face $O$. With no loss in generality, let $p_1 < p_2 < p_3 < \overline{p}$. The pricing equations in (3) are still valid, and for the three firms to be on the same OBSF iso-face value curve, it must be that $Y_L^1 < Y_L^2 < Y_L^3$.

Using expressions (1), (2) and (3), the BSF and the OBSF face values for firm $i$ ($i = 1, 2, 3$) are

$$\begin{aligned} B(p; r_i) &= \frac{I}{(1 - p_i\left[p_i + r_i(1 - p_i)\right])}, \forall Y_L, \text{ and} \\ O &= \frac{I}{(1 - p_i)} - \frac{p_i Y_L^i}{(1 - p_i)}, \ i = 1, 2, 3. \end{aligned} \tag{8}$$

Our earlier results suggest BSF would be most attractive to Firm 1, the least risky firm. Suppose Firm 1 does use BSF and selects $r = 1$; the face value of BSF debt for Firm 1 would be

$$B(p_1; 1) = \frac{I}{(1 - p_1)}.$$

Firm 2, a riskier firm, can affect the face value of its BSF debt by diversifying; selecting a new project with a smaller $r$ reduces the face value of its BSF debt. In particular, by choosing $r_2 = \frac{p_1 - (p_2)^2}{(1 - p_2)p_2} < 1$, Firm 2 can get the same face value for its BSF debt as Firm 1, since

$$B\left(p_2; r_2 = \frac{p_1 - (p_2)^2}{(1 - p_2)p_2}\right) = \frac{I}{(1 - p_1)} = B(p_1; 1).$$

While choosing $r_2 < 1$ makes debt cheaper for Firm 2, it has costly cash flow effects, as discussed in Proposition 1. However, if outsiders are not able to differentiate the degrees to which Firm 1 and Firm 2 have chosen to diversify, there is an offsetting valuation benefit. As long as Assumption 6 is satisfied, and $(1 - r_2) < k$, shareholders will not be able to separate Firm 2 from Firm 1 and will value their stocks at the same price.

Firm 3 might try a similar strategy, diversifying extensively in an attempt to pool with Firms 1 and 2. By selecting $r_3 = \frac{p_1 - (p_3)^2}{(1-p_3)p_3} < r_2$, the face value of the BSF debt for Firm 3 is the same as that for Firms 1 and 2. However, if the risk characteristics of Firm 3 are significantly different from Firm 1, and Assumption 6 is violated, this approach would not work. Firm 3 would be identified because the diversification associated with its project is so different from $r = 1$ selected by Firm 1. If Firm 3 cannot use BSF without being identified, it is better off using OBSF by itself and being identified.

Proposition 4 below relies on these arguments to identify an equilibrium in which both BSF and OBSF emerge even when the correlation parameter is only partially observable.

**Proposition 4** *Let $r$ be a choice variable. Given assumptions 1-6, there exist parameter values supporting a sequential equilibrium in which firms 1 and 2 pool by choosing BSF and Firm 3 selects OBSF. Firm 1 selects $r_1 = 1$ , Firm 2 selects $r_2 = \frac{p_1 - (p_2)^2}{(1-p_2)p_2} < 1$.*

The proof in the appendix identifies three parametric conditions for the existence of this equilibrium, and presents a numerical example which establishes that these conditions are met for a range of parameter values. Thus, the basic result that valuation considerations can influence some firms to choose the costlier BSF alternative holds when the extent of diversification achieved by the adoption of the project is only partially observable. In fact, this lack of observability results in some firms choosing costly levels of diversification; the diversification costs are more than offset by valuation gains achieved by pooling with lower risk firms.

Finally we note if differences in diversification were observable, some firms would revert to OBSF.

**Corollary**: *For some parameter values, there exists a set of firms that chooses BSF and diversification when there are limits on the observability of $r$, and choose OBSF when $r$ is observable.*

The proof identifies a parametric condition guaranteeing all three firms prefer OBSF to BSF in equilibrium. Using the same numerical example as in Proposition 4, we show the range of parameter values that meet this condition overlaps with the range that meets the conditions of Proposition 4. Together, Proposition 4 and the Corollary imply the accounting treatment/financing choice is affected by the degree to which diversification can be observed. If there are limits on firms' attempts to imitate less risky firms, then one would expect to observe more OBSF.

## 4 Conclusion

In this paper we show that informational issues can override financing costs when a firm decides what project to undertake and the mode of debt financing. In choosing between additional conventional debt (BSF) and project/off-the-balance sheet financing (OBSF) for a new project, a firm with some debt already in its capital structure weighs increased expected cash flows for shareholders with OBSF versus increased information about the firm's risk characteristics with BSF. We find that less risky firms have less to gain with OBSF and have better news to provide the stock market with BSF; hence, they select BSF. We also study the impact of the firm's project selection decision on this result. In particular, if the firm can choose the correlation between the cash flows on the project and the cash flows from existing assets, and if shareholders can only imperfectly observe this correlation, BSF can become attractive to even riskier firms (firms that would choose OBSF if the correlation were perfectly observable). By diversifying and using BSF, such a firm may be able to pool itself with other firms which are less risky than itself.

In the paper, firms' financing choices have been limited to conventional debt and project financing. We have not considered equity and self-financing. In our model, with equity financing no information is supplied to the market about the firm's type except the fact that the firm issued new equity to undertake a new project. If the firm used its own funds for the project, the financing would be cheaper than equity financing, but again no information about the firm or the project would be revealed other than the fact that an internally funded project was taken. Thus, as long as firms have their own funds to invest in the project, there is no demand for equity financing in our setting. Which firms would use self-financing as opposed to BSF or OBSF is an equilibrium question beyond the scope of this paper. Since our focus is the choice between on- and off-balance sheet debt we have considered only debt financing, and the information that is revealed by this choice.

# Appendix A

## Overview of Project Financing

A popular off-balance sheet financing method is project financing. In a typical project financing transaction two or more firms, sharing their need for a common factor input, form a joint venture to invest in the required assets. A legally separate firm is formed with capital being raised through the sponsoring firms' commitments to purchase the project's output. Examples of project financing transactions include *take-or-pay* and *throughput contracts*. Thus, project financing is a way of acquiring *production capacity* without *owning* the project (Laibstein, Stout & Bailey [1988]). An example of a project financing transaction can be found in the February 1982 issue of *Dun Business Month* in the Money and Markets section. In this instance, a leveraged lease was arranged for Chicago & North Western Transportation Co. and Union Pacific Corporation to construct a rail connection. Fifty percent of the outlay of $460 million was raised from a consortium of banks. A 'trend-setting' feature of the deal was that banks will not get their money back if the new railroad does not produce enough traffic to pay off the debt. For some more examples of off-balance sheet financing, see Toll [1996].

From a legal perspective, since the company does not own the project, property rights restrict existing debtholders' seniority claims. Since the project is a legally separate entity, debtholders cannot insist in their debt contracts that they have senior claim to the projects' assets. It is conceivable that debtholders could write into the debt contract a restriction on the company's ability to invest in projects or companies that were funded by debt. It is not clear, however, whether their interests would not be served by such a restriction. While the lenders of new capital for the project have first lien on the project's assets, they have no recourse to the sponsoring company's existing assets. Thus, the interests of original debtholders in the company are protected in the event that the project were to fail, and the original debt holders have nothing to gain from limiting the company's use of project financing methods (Hoffman [1989b]).

From an accounting perspective, such transactions stay off the balance sheet. The companies sponsoring the project typically hold minority interests so that consolidation is not required. In addition, while the firms sponsoring the project agree to make payments for some minimum output from the project, such unconditional purchase obligations are not recorded as a liability because they are essentially executory contracts for future goods and services. Thus, by current accounting practices, the decision to use project financing is linked to the decision to list the project off the balance sheet.

Finally, because project financing (off balance sheet financing) allows the company to avoid the current debtholders' subordination clauses, it is typically cheaper to finance the project off the balance sheet. Hoffman [1989a] writes that a project financing is selected in many circumstances because more attractive interest rates and credit enhancement are available to the project rather than are otherwise available to the project sponsor. Nevertheless we show that it is in some firms' interests to not use project financing, but to opt instead for conventional (subordinate) debt.

## Appendix B

Computation of state probabilities as a function of $p$ and $r$

The four probabilities, $\delta_{HH}$, $\delta_{HL}$, $\delta_{LH}$, and $\delta_{LL}$ must satisfy the following set of equations:

$$\begin{aligned}
&\delta_{HH} + \delta_{HL} + \delta_{LH} + \delta_{LL} = 1, \\
&\delta_{HH} + \delta_{HL} = \Pr(X_H) = 1 - p, \\
&\delta_{HH} + \delta_{LH} = \Pr(Y_H) = 1 - p, \\
&r = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}.
\end{aligned}$$

Solving these equations yields the expressions for $\delta_{HH}$, $\delta_{HL}$, $\delta_{LH}$, and $\delta_{LL}$ as stated in (1). ■

Proof that $p \leq \overline{p}(r)$ insures $O(Y_L, p) < Y_H$.

From pricing equation (3), $O(Y_L, p) < Y_H$ iff $p < \frac{Y_H - I}{Y_H - Y_L}$. From Assumption 5 and equation (2), $\overline{p}(r)$ is the solution to

$$X_L + Y_H - F = \frac{1}{1 - \delta_{Ll}(\overline{p}(r); r)}.$$

Using the expression for $\delta_{Ll}(\overline{p}(r); r)$ from equation (1) and differentiating:

$$\frac{d\overline{p}(r)}{dr} = -\frac{(1-p)p}{2p(1-r) + r} < 0.$$

Hence $\overline{p}(r)$ is the smallest when $r = 1$. At $r = 1$,

$$\overline{p}(1) = \frac{X_L + Y_H - F - I}{X_L + Y_H - F}.$$

Note $\frac{Y_H - I}{Y_H - Y_L} - \frac{X_L + Y_H - F - I}{X_L + Y_H - F} = \frac{(I - Y_L)(F - X_L - Y_L) + Y_L(Y_H - Y_L)}{(Y_H - Y_L)(X_L + Y_H - F)} > 0$, $\frac{Y_H - I}{Y_H - Y_L} > \overline{p}(r)$ for all $r$. Therefore, whenever $p < \overline{p}(r)$ for all $r$, $p < \frac{Y_H - I}{Y_H - Y_L}$, which insures $O(Y_L, p) < Y_H$. ■

Proof of Observation 2

When $r = 1$, the probability of bankruptcy $\delta_{LL}(p; 1) = p$, and therefore from (2),

$$B(p; 1) = \frac{I}{1 - p}.$$

From (3),

$$O(Y_L, p) = \frac{I}{(1-p)} - \frac{pY_L}{(1-p)}.$$

Comparing the above two equations $O(Y_L, p) \leq B(Y_L, p; 1)$, with the inequality being strict for strictly positive values of $p$ and $Y_L$.

When $0 < p < Min[.5, \overline{p}]$, and $r = r_{\min} = -\frac{p}{1-p}$,

$$B(Y_L, p; r_{\min}) = I.$$

Hence $O(Y_L, p) \geq B(Y_L, p; r_{\min})$, with the inequality being strict for $Y_L < I$.

When $.5 < p < \overline{p}$, and $r = r_{\min} = -\frac{1-p}{p}$,

$$B(Y_L, p; r_{\min}) = \frac{I}{2(1-p)}.$$

By comparing the two expressions for the face value of the debts, $O(Y_L, p) \geq B(Y_L, p; r_{\min})$ when $p \leq \frac{I}{2Y_L}$. ■

Proof of Proposition 1

Referring to equation (4), the derivative of the expected profit $\pi_S^B(Y_L, p; r)$ with respect to $r$ is

$$\begin{aligned}
\frac{d\pi_S^B(Y_L, p; r)}{dr} \quad = \quad & \frac{\partial \delta_{HH}(p; r)}{\partial r}\left[X_H + Y_H - F - B(p; r)\right] + \\
& \frac{\partial \delta_{HL}(p; r)}{\partial r}\left[X_H + Y_L - F - B(p; r)\right] + \\
& \frac{\partial \delta_{LH}(p; r)}{\partial r}\left[X_L + Y_H - F - B(p; r)\right] + \\
& - \left[\delta_{HH}(p; r) + \delta_{HL}(p; r) + \delta_{LH}(p; r)\right]\frac{\partial B(p; r)}{\partial r}. \qquad (9)
\end{aligned}$$

From (1) and (2),

$$\begin{aligned}
\frac{\partial \delta_{HH}(p; r)}{\partial r} &= p(1-p), \\
\frac{\partial \delta_{HL}(p; r)}{\partial r} &= -p(1-p),
\end{aligned}$$

$$\frac{\partial \delta_{LH}(p;r)}{\partial r} = -p(1-p),$$
$$\delta_{HH}(p;r) + \delta_{HL}(p;r) + \delta_{LH}(p;r) = 1 - \delta_{LL}(p;r),$$
$$\frac{\partial B(p;r)}{\partial r} = \frac{I}{(1-\delta_{LL}(p;r))^2} p(1-p).$$

Substituting these expressions into (9), using a little algebra, and noting that $F > X_L + I \geq X_L + Y_L$,

$$\frac{d\pi_S^B(Y_L, p; r)}{dr} = p(1-p)\left[F - X_L - Y_L\right] > 0.$$

Because the expected profit is increasing in $r$, and the manager is assumed to act in the best interests of the shareholders, the firm will choose $r = 1$. The rest of the Proposition follows from Observation 2, and from the fact that diversification is not relevant when OBSF is chosen. ■

Proof of Observation 3

With $r$ observable, shareholders can infer $p$ perfectly from the pricing equation (2). Therefore, the expected profit with balance sheet financing for a firm $(p, Y_L)$ can be written

$$\begin{aligned} E^e\left[\pi_S^B|B\right] &= \delta_{HH}(p;r)\left[X_H + Y_H - F - B\right] + \\ &\quad \delta_{HL}(p;r)\left[X_H + E^e[Y_L|B] - F - B\right] + \\ &\quad \delta_{LH}(p;r)\left[X_L + Y_H - F - B\right], \end{aligned} \tag{10}$$

where $E^e$ is the expectation operator for some equilibrium. The derivative of the expected profit with respect to $r$ is

$$\frac{dE^e\left[\pi_S^B|B\right]}{dr} = p(1-p)\left[F - X_L - E^e[Y_L|B]\right]$$

Now, in any equilibrium $E^e[Y_L|B] \leq I$ because $I$ is the upper bound on $Y_L$. Further, because $F > X_L + I$ from Assumption 2, it follows that $\frac{dE^e\left[\pi_S^B|B\right]}{dr} > 0$, which implies that the firm will choose $r = 1$. ■

Proof of Proposition 2

First, we demonstrate all firms will undertake the project. Suppose $N$ is the equilibrium set of all firms who choose no project, and let $p_N$ be the smallest value of $p$ in that set. If the firm with $p = p_N$ does not take the project, its expected profits are:

$$\pi^N = (1 - E[p|N])[X_H - F].$$

where $E[p|N] > p_N$. If the firm uses BSF, $p_N$ is revealed, and its expected profits are:

$$\pi^B = (1 - p_N)\left[X_H + Y_H - F - B\right].$$

Since $E[p|N] \geq p_N$ and, by assumption 5, $Y_H > B$, the firm would never forego the project. If $p_N$ does not exist, since $Y_H$ is strictly larger than $B$, and the payoffs are continuous in $p$, a limiting argument gives the same result.

Next, consider two firms $(Y_{L1}, p_1)$ and $(Y_{L2}, p_2)$ on the same OBSF iso-face-value curve, or $O(Y_{L1}, p_1) = O(Y_{L2}, p_2)$, where $p_1 < p_2$. We show if firm $(Y_{L1}, p_1)$ is indifferent between OBSF and BSF, then in equilibrium firm $(Y_{L2}, p_2)$ strictly prefers OBSF to BSF. To see why,

$$\begin{aligned}
\pi^B(Y_{L2}, p_2) &= (1 - p_2)\left(X_H + Y_H - F - \frac{I}{1 - p_2}\right) \\
&< \pi^B(Y_{L1}, p_1) = (1 - p_1)\left(X_H + Y_H - F - \frac{I}{1 - p_1}\right) \\
&= \pi^O(Y_{L1}, p_1) = (1 - E\left[p|O(Y_{L1}, p_1)\right]\left[X_H + Y_H - F - O(Y_{L1}, p_1)\right] \\
&= \pi^O(Y_{L2}, p_2) = (1 - E\left[p|O(Y_{L2}, p_2)\right]\left[X_H + Y_H - F - O(Y_{L2}, p_2)\right].
\end{aligned}$$

We now demonstrate in any equilibrium the least risky firms select BSF. Consider the firm $(Y_L, 0)$, $Y_L \in [0, I)$. If this firm uses BSF, its profits are $\pi^B(Y_L, 0) = X_H + Y_H - F - I$. Note this is the maximum profits any firm with any financing method can earn. If the firm $(Y_L, 0)$ were to use OBSF, it could earn the same profits only if $E[p|O(Y_L, 0)] = 0$. But if firm $(Y_L, 0)$ is indifferent, then firm $(I, p)$, $0 < p \leq \overline{p}$, which lies on the same OBSF iso-face-value curve as $(Y_L, 0)$, strictly prefers OBSF. Hence $E[p|O(Y_L, 0)] = 0$ is not a consistent belief. Next, for each $Y_L$, consider the firm $(Y_L, \varepsilon)$, where $Y_L \in [0, I)$. By a similar argument, in any equilibrium, this firm would choose BSF for a small enough $\varepsilon$.

Finally, we demonstrate in any equilibrium, the riskiest firms select OBSF. Consider the firm $(Y_L, \overline{p})$ where $Y_L \in (0, I]$. If this firm uses BSF,

$$\pi^B(Y_L, \overline{p}) = (1 - \overline{p})\left(X_H + Y_H - F - \frac{I}{1 - \overline{p}}\right)$$

If the firm uses OBSF, since $E[p|O(Y_L,\overline{p})] \leq \overline{p}$,

$$
\begin{aligned}
\pi^O(Y_L,\overline{p}) &= (1 - E[p|O(Y_L,\overline{p})]) \left( X_H + Y_H - F - \left[ \frac{I}{1-\overline{p}} - \frac{\overline{p}Y_L}{1-\overline{p}} \right] \right) \\
&\geq (1-\overline{p}) \left( X_H + Y_H - F - \left[ \frac{I}{1-\overline{p}} - \frac{\overline{p}Y_L}{1-\overline{p}} \right] \right) \\
&> (1-\overline{p})X_H + Y_H - F - \frac{I}{1-\overline{p}} \\
&= \pi^B(Y_L,\overline{p})
\end{aligned}
$$

Thus OBSF is strictly preferred in any equilibrium. Now, for each $Y_L$, consider the firm $(Y_L, \overline{p} - \varepsilon)$ where $Y_L \in (0, I]$. By a similar argument, for small enough $\varepsilon$, in any equilibrium, this firm would choose OBSF.

Proof of Propositions 3

We show there exists a boundary $p^*(Y_L)$ such that all firms $(Y_L, p)$ with $0 \leq p < p^*(Y_L)$ will choose BSF, and $(Y_L, p)$ with $p^*(Y_L) < p \leq \overline{p}$ will choose OBSF. All firms on the boundary are indifferent between choosing BSF and OBSF. There is a positive measure of firms using BSF and OBSF in the equilibrium.

For a given firm $(Y_L, p)$, define

$$
\Delta\pi(Y_L,p) \equiv \pi^B(Y_L,p) - \pi^O(Y_L,p),
$$

where

$$
\begin{aligned}
\pi^B(Y_L,p) &= (1-p)\left[ X_H + Y_H - F - \frac{I}{(1-p)} \right], \\
\pi^O(Y_L,p) &= (1 - E\left[p|O(Y_L,p)\right]) \left[ X_H + Y_H - F - \left( \frac{I}{(1-p)} - \frac{pY_L}{(1-p)} \right) \right],
\end{aligned}
$$

and $E\left[p|O(Y_L,p)\right]$ is the expectation operator over all equally risky or riskier firms lying on the same iso-face value OBSF curve. The term $\pi^B(Y_L,p)$ represents the perceived value of the firm if it chooses BSF, and the term $\pi^O(Y_L,p)$ represents its perceived value if all equally risky or riskier firms on the same OBSF iso-face value curve opt for OBSF. The difference $\Delta\pi(Y_L,p)$ is clearly continuous on the domain $[0,I] \times [0,\overline{p}]$.

Loosely, the proof is established by showing on the border of the space $[0,I] \times [0,\overline{p}]$ the difference $\Delta\pi(Y_L,p)$ takes on the values as indicated in parentheses in Figure 3. Given these values for the border, and using a fixed point theorem by Browder [1960], we show a boundary can be drawn separating firms using OBSF from those using BSF, as pictured in figure 3.

(Figure 3 here)

Formally, along the border of the space $[0, I] \times [0, \overline{p}]$ we observe the difference $\Delta\pi(Y_L, p)$ takes on the following values:

- $\Delta\pi(0, p) > 0$ for $p < \overline{p}$.
  Because $O(0, p) = B(0, p) = \frac{I}{1-p}$, this firm is indifferent from a financing standpoint. On the other hand, $E[p|O(0, p)] > p$, or if the firm chooses OBSF, it will be perceived as a riskier firm than it actually is because it will be grouped with other firms on the same OBSF iso-face value curve that are at least as risky as itself. The two profit functions are as follows:

$$\begin{aligned} \pi^B(0, p) &= (1-p)\left[X_H + Y_H - F - \frac{I}{1-p}\right], \text{ and} \\ \pi^O(0, p) &= (1 - E[p|O(0, p)])\left[X_H + Y_H - F - \frac{I}{1-p}\right], \end{aligned}$$

making it clear that $\Delta\pi(Y_L, p) > 0$ .

- $\Delta\pi(0, \overline{p}) = 0.$

Because all OBSF iso-face value curves slope upward, the firm $(0, \overline{p})$ is the only firm on its iso-face-value curve. Hence upon observing that the firm has chosen OBSF, and that the OBSF debt $O(0, \overline{p}) = \frac{I}{1-\overline{p}}$, outsiders will correctly perceive this firm to be of type $(0, \overline{p})$. If this firm were to choose BSF, then $B(0, \overline{p}) = \frac{I}{1-\overline{p}}$, and once again, outsiders infer $\overline{p}$ correctly. Therefore, the firm $(0, \overline{p})$ is indifferent between choosing OBSF and BSF because financing cost as well as firm valuation are the same with both options.

- $\Delta\pi(Y_L, \overline{p}) < 0$, for $0 < Y_L \leq I$.
  This is immediate from Proposition 2.

- $\Delta\pi(Y_L, 0) > 0$, for $0 < Y_L \leq I$.
  This is immediate from Proposition 2.

- There exists a unique $p_i, 0 < p_i < \overline{p}$, such that $\Delta\pi(I, p_i) = 0$ and $\Delta\pi(I, p) < (>)0$ for $p > (<)p_i$.
  Notice that $E[p|O(I,p)] = \frac{p+\overline{p}}{2}$. Further, $O(I,p) = I$, and $B(I,p) = \frac{I}{1-p}$. Therefore,

$$\begin{aligned}\Delta\pi(I,p) &= (1-p)\left[X_H + Y_H - F - \frac{I}{(1-p)}\right] - \\ &\quad (1 - \frac{p+\overline{p}}{2})\left[X_H + Y_H - F - I\right].\end{aligned}$$

- It can be easily shown that $\Delta\pi(I,p)$ is strictly decreasing in $p$, $\Delta\pi(I,0) > 0$, and $\Delta\pi(I,\overline{p}) < 0$. Therefore, there exists a unique $p_i$ as described above.

Define the set $S = \{(Y_L, p)\,|\Delta\pi(Y_L, p) = 0\}$. Given the properties of the function $\Delta\pi(Y_L, p)$ established above, it follows from Browder [1960]that there exists in the set $S$ a simply connected path going from $(0, \overline{p})$ to $(I, p_i)$. Let $\widehat{S}$ be the set of points on that path.

We can now construct an equilibrium as follows. On each OBSF iso-face value curve $p(Y_L; O_1)$, let $Y_L^*$ be the largest value of $Y_L$ such that $(Y_L^*, p(Y_L^*; O_1)) \in \widehat{S}$. (The OBSF iso-face value curve going through the single point $(0, \overline{p})$ consists only of that point; because $\Delta\pi(0, \overline{p}) = 0$, the point $(0, p(0; O_1))$ is in $\widehat{S}$.) Then, $Y_L^*$ is well defined, and lies in the "middle" of the OBSF curve. Assign those firms on the OBSF iso-face value curve above $Y_L^*$ to OBSF, and the rest to BSF. This is an equilibrium because no firm will wish to deviate from its type of financing. The firms on the boundary are indifferent between the two types of financing. This construction is illustrated in Figure 4. By construction of the boundary there is a positive measure of firms selecting BSF and OBSF in equilibrium. ■

Proof of Proposition 4

The conjectured equilibrium strategies are:

*Firm 1: Chooses BSF and selects* $r_1 = 1$.

*Firm 2: Chooses BSF and selects* $r_2 = \frac{p_1 - (p_2)^2}{(1-p_2)p_2} < 1$.

*Firm 3: Chooses OBSF.*

To insure that $r_2$ exists, or $r_2 > r_{\min}$, we note if $p_2 < .5$, then $r_2 > r_{\min}$ for all $p_1$; if $p_2 > 0.5$, then $r_2 > r_{\min}$ if

$$p_1 > 2p_2 - p_1. \tag{11}$$

The BSF face value for Firms 1 and 2 can be computed using equation (8) as $B(p_1; 1) = B(p_2; r_2) = \frac{I}{(1-p_1)}$. To ensure that outsiders cannot differentiate between Firm 1 and 2 in this conjectured equilibrium, we require that $1 - r_2 < k$. With some algebra this condition translates into the following condition on $p_1$:

$$p_1 > p_2 \left[1 - k\left(1 - p_2\right)\right]. \tag{12}$$

To ensure that Firm 3 cannot pool with Firm 2 by choosing $r_3 = \frac{p_1-(p_3)^2}{(1-p_3)p_3}$ generating $B(p_3; r_3) = \frac{I}{1-p_1}$, we require that $r_2 - r_3 > k$. Solving for $r_3$, we get the following condition:

$$\text{If } 1 - p_2 - p_3 > (<)0, \text{ then } p_1 > (<)\frac{p_2 p_3 \left[(p_2 - p_3) + k(1-p_2)(1-p_3)\right]}{(p_3 - p_2)\left(1 - p_2 - p_3\right)}. \tag{13}$$

Note the above condition ensures that $1 - r_3 > k$ as well. Firm 1's (and Firm 2's) expected profit under the equilibrium strategy is $\pi^1(BSF) = \frac{1}{2}\left[\pi_S^B(Y_L^1, p_1; 1) + \pi_S^B(Y_L^2, p_2; r_2)\right]$, where $\pi_S^B(Y_L, p; r)$ is defined in equation (4) in the text. If Firm 1 or Firm 2 were to deviate and choose OBSF, it will be identified as Firm 3, and its valuation would be $\pi^1(OBSF) = (1-p_3)[X_H + Y_H - F - O]$. Thus we require $\pi^1(BSF) > \pi^1(OBSF)$, or the condition:

$$\begin{aligned} & p_1 \left[\frac{I - O(1-p_2) + p_2(X_L + X_H + Y_H - 2F)}{2p_2}\right] < \\ & \frac{1}{2}\left\{\begin{array}{c} -I + O(1 + p_2 - 2p_3) + \\ 2p_3(X_H + Y_H - F) - p_2(X_H - X_L + Y_H) \end{array}\right\}. \end{aligned} \tag{14}$$

Next, we want to insure that it is not possible for Firm 1 to select a diversification level which would prohibit Firm 2 from imitating Firm 1's risk characteristics. If Firm 1 selected $r_1 < 1$, Firm 2 will get BSF with the same face value if

$$r_2 = \frac{p_1^2(1-r_1) + p_1 r_1 - p_2^2}{(1-p_2)p_2}. \tag{15}$$

The difference $r_1 - r_2$ is increasing in $r_1$ if $1 - \frac{(1-p_1)p_1}{(1-p_2)p_2} > 0$, or $(1-p_1) > p_2 > p_1$. Thus the condition $p_1 < (1 - p_2)$ is sufficient but not necessary to insure that Firm 1 cannot separate itself. Finally, because Firm 3 cannot pool with Firms 1 and 2, its best option is to choose OBSF because it is the cheaper mode of financing.

It is assumed off-the-equilibrium path, outsiders believe a firm with BSF, whose face value is other than $\frac{I}{1-p_1}$ is Firm 2 with probability one. These beliefs, together with conditions (11)-(15), determine a sequential equilibrium which satisfies the intuitive criterion.

Numerical example

In Figure 4, we present a numerical example to illustrate the existence of the above equilibrium for a reasonable set of parameter values. In particular, it shows the ranges of values for $p_1$ and $p_2$ for which the equilibrium conditions are satisfied.

(Figure 4 here)

■

Proof of Corollary

As demonstrated previously, if $r$ is observable, $r = 1$ is preferred by all firm types. All three firms prefer OBSF when

$$\pi_S^B(Y_L^1, p_1; 1) < \frac{1}{3}\left[\pi_S^O(Y_L^1, p_1) + \pi_S^O(Y_L^2, p_2) + \pi_S^O(Y_L^3, p_3)\right].$$

This condition is met when

$$p_1 > \frac{(p_2 + p_3)\left[X_H + Y_H - O - F\right] + 3(O - I)}{2\left[X_H + Y_H - O\right] + O}.$$

Figure 5 illustrates that the set of parameters satisfying this condition overlaps with the set of parameters satisfying the equilibrium condition in Proposition 4.

(Figure 5 here)

■

## References


Berkovitch, E. and E.H. Kim (1990). Financial Contracting and Leverage Induced Over- and Under-Investment Incentives. *The Journal of Finance*, **XLV**, 765–794.

Browder, F. (1960). On Continuity of Fixed Points Under Deformations of Continuous Mappings. *Summa Brasiliensis Mathematica*, 183–191.

Dye, R. A. and R. E. Verrecchia (1995). Discretion vs Uniformity: Choices Among GAAP. *The Accounting Review*, **70**, 389–415.

Hoffman, S. (1989a). Loans Based on Cash Flow and Contracts. *Commercial Lending Review*, **4**, 18–30.

Hoffman, S. (1989b). A Practical Guide to Transactional Project Finance: Basic Concepts, Risk Identification, and Contractual Considerations. *Business Lawyer*, **45**, 181–232.

James, C. (1989). Off-Balance Sheet Activities and the Underinvestment Problem in Banking. *Journal of Accounting, Auditing and Finance*, **4**, 111–124.

Laibstein, S., D. E. Stout, and L. P. Bailey (1988). Managing Off-Balance Sheet Financing. *Management Accounting*, 32–39.

Levine, C. (1996). Information Revelation and Accounting for Stock Based Compensation. Technical report, Carnegie Mellon University.

Nevitt (1979). *Project Financing*. Euromoney Publications.

Rajan, R. (1992). The Choice between Informed and Arm's Length Debt. *The Journal of Finance*, **47**, 1367–1400.

Rajan, R. and A. Winton (1995). Covenants and Collateral as Incentives to Monitor. *The Journal of Finance*, **50**, 1113–1146.

Shah, S. and A. Thakor (1987). Optimal Capital Structure and Project Financing. *Journal of Economic Theory*, **42**, 209–243.

Toll, B. (1996). Capital Off the Books. *Oil & Gas Investor*, **16**, 41–43.

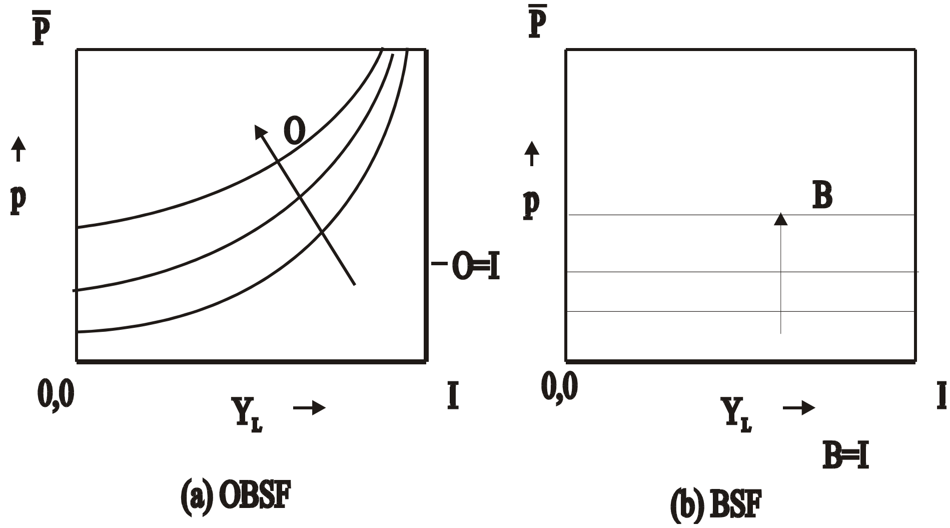


Figure 1:

**Figure 1: Iso-Face Value Curves**

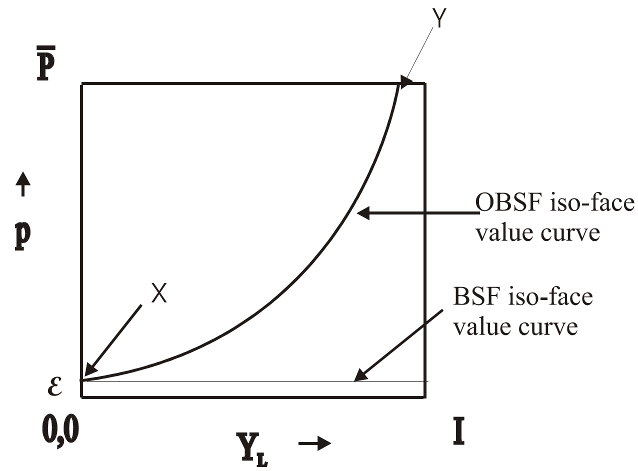


Figure 2:

**Figure 2: Financing Method and the Firm's Risk**

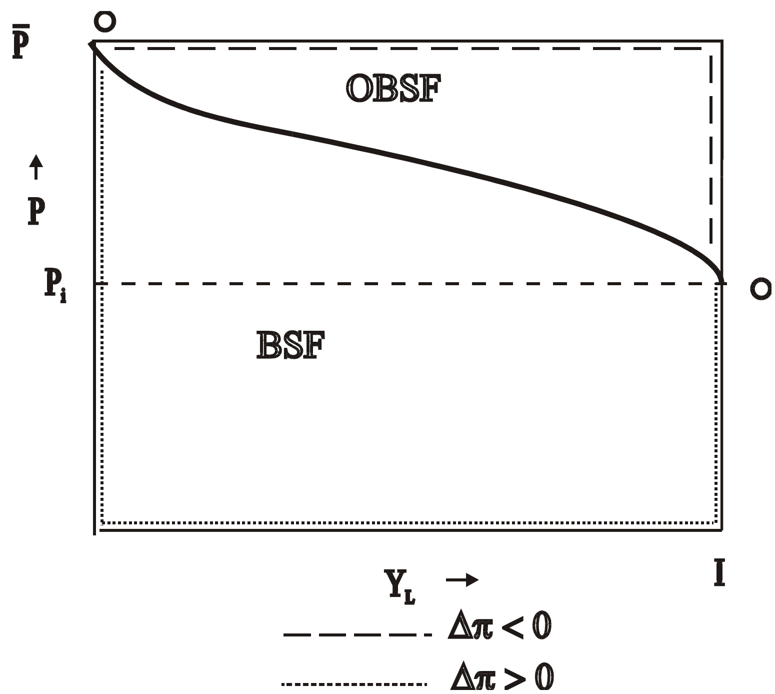


Figure 3:

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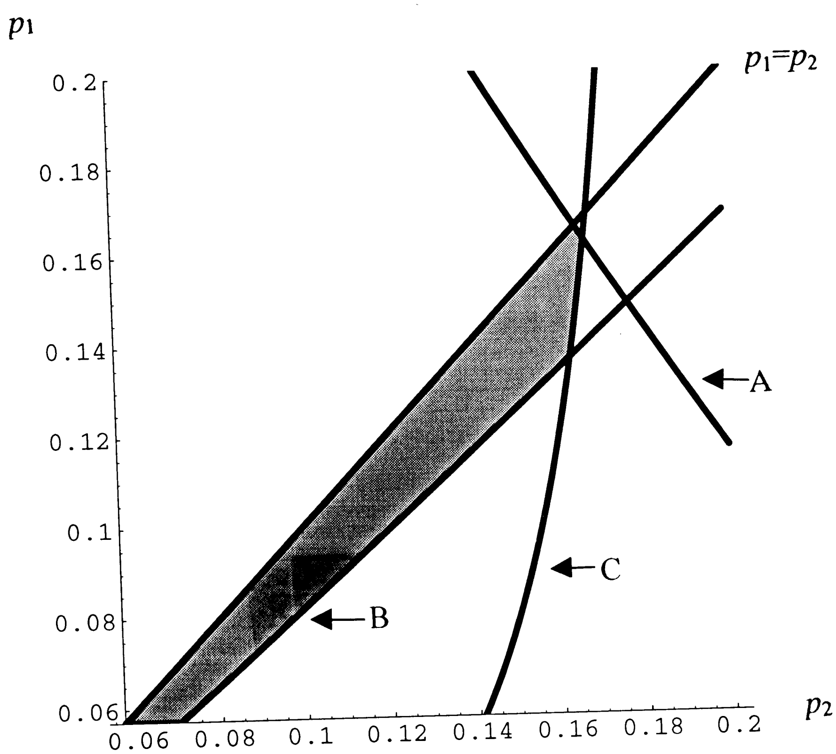
**Figure 3: BSF amd OBSF Regions**




Parameter values $Y_H = 35, X_H = 50, X_L = 10, I = 14, F = 30, p_3 = 0.2, k = 0.2$ and $O = 15$.

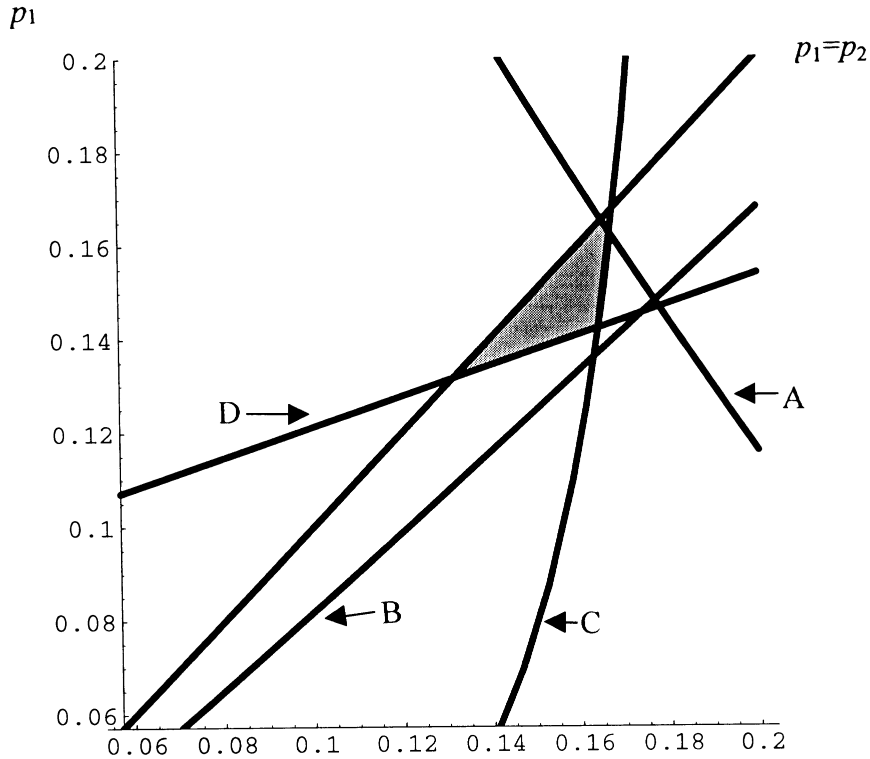
$p_1 < p_2$ insures that Firm 1 is the least risky firm.

$p_1 < A$ insures that Firm 1 prefers to be pooled with Firm 2 using BSF rather than use OBSF and be perceived as Firm 3.

$p_1 > B$ insures that $r_2$ cannot be differentiated from $r_1 = 1$, or that Firms 1 and 2 cannot be distinguished through the diversification of their projects.

$p_1 > C$ insures that Firm 3 cannot pool with Firms 1 and 2.

**Figure 4: Graph illustrating Proposition 4**




Parameter values $Y_H = 35, X_H = 50, X_L = 10, I = 14, F = 30, p_3 = 0.2, k = 0.2$ and $O = 15$.

$p_1 < p_2$ insures that Firm 1 is the least risky firm.

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$p_1 > B$ insures that $r_2$ cannot be differentiated from $r_1 = 1$, or that Firms 1 and 2 cannot be distinguished through the diversification of their projects.

$p_1 > C$ insures that Firm 3 cannot pool with Firms 1 and 2.

$p_1 > D$ insures that Firms 1, 2 all prefer OBSF to BSF.

**Figure 5: Graph illustrating Corollary to Proposition 4**