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Multiple-criteria decision-making in two-sided assembly line balancing: A goal programming and a fuzzy goal programming models

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ABSTRACT

Two-sided assembly lines are especially used at the assembly of large-sized products, such as trucks and buses. In this type of a production line, both sides of the line are used in parallel. In practice, it may be necessary to optimize more than one conflicting objectives simultaneously to obtain effective and realistic solutions. This paper presents a mathematical model, a pre-emptive goal programming model for precise goals and a fuzzy goal programming model for imprecise goals for two-sided assembly line balancing. The mathematical model minimizes the number of mated-stations as the primary objective and it minimizes the number of stations as a secondary objective for a given cycle time. The zoning constraints are also considered in this model, and a set of test problems taken from literature is solved. The proposed goal programming models are the first multiple-criteria decision-making approaches for two-sided assembly line balancing problem with multiple objectives. The number of mated-stations, cycle time and the number of tasks assigned per station are considered as goals. An example problem is solved and a computational study is conducted to illustrate the flexibility and the efficiency of the proposed goal programming models. Based on the decision maker's preferences, the proposed models are capable of improving the value of goals.

1. Introduction

Assembly lines are usually designed in a range of industries to produce a high-volume standardized homogeneous products. An assembly line comprises a series of successive stations connected together by a material handling system in which the components are consecutively assembled into a final product and they are moved from one station to its successive station until they reach to the end of the line. The components are processed depending on a set of tasks, and they are performed at each station during a fixed time called as cycle time. The tasks are allocated to stations according to given precedence relationships among tasks and specific restrictions which aim to optimize one or more objectives, such as minimizing the number of stations for a given cycle time or minimizing the cycle time for a given number of stations. A feasible assignment of tasks to stations should guarantee that the following constraints: (i) each task must be assigned to exactly one station (the assignment constraint), (ii) all precedence relationships among tasks must be satisfied (the precedence constraint) and (iii) the total task times of all the tasks assigned to a station cannot exceed the cycle time (the cycle time constraint). The problem of assigning tasks to stations in such a way that one or more objectives are optimized subject to some specific restrictions is called as the assembly line balancing problem (ALBP).

In the literature, generally, the studies on assembly lines are classified as; straight (traditional) assembly lines, and U-lines (U-shaped assembly lines) by means of the line layout, and also it is classified as means of the number of product models produced on the line; single-model, and mixed/multi-model lines. For more details on classification of assembly lines refer to Boysen et al. [1]. Many studies on assembly lines including exact solution methods, heuristics, and meta-heuristic approaches have been reported in the literature. The detailed reviews of such studies are given at Baybars [2], Ghosh and Gagnon [3], Erel and Sarin [4], and more recently by Scholl and Becker [5], and Becker and Scholl [6].

Assembly lines can also be categorized as one-sided assembly lines and two-sided assembly lines. The difference between them is associated with the design of the line, i.e., in the two-sided assembly lines, the left-side and the right-side of the line are used in parallel, the operators working in opposite sides of the line can perform their tasks on the same component simultaneously, whereas only a specific side of the line is used in the one-sided assembly lines. Two-sided assembly lines are typically found in assembling large-sized

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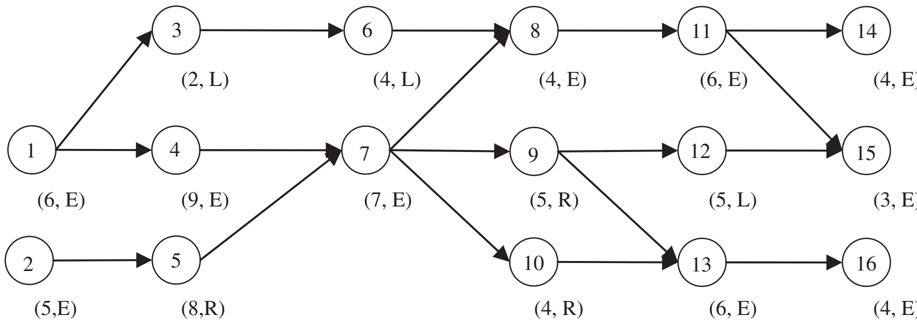

Fig. 1. Precedence diagram, task times and preferred operation directions of the 16-task problem.

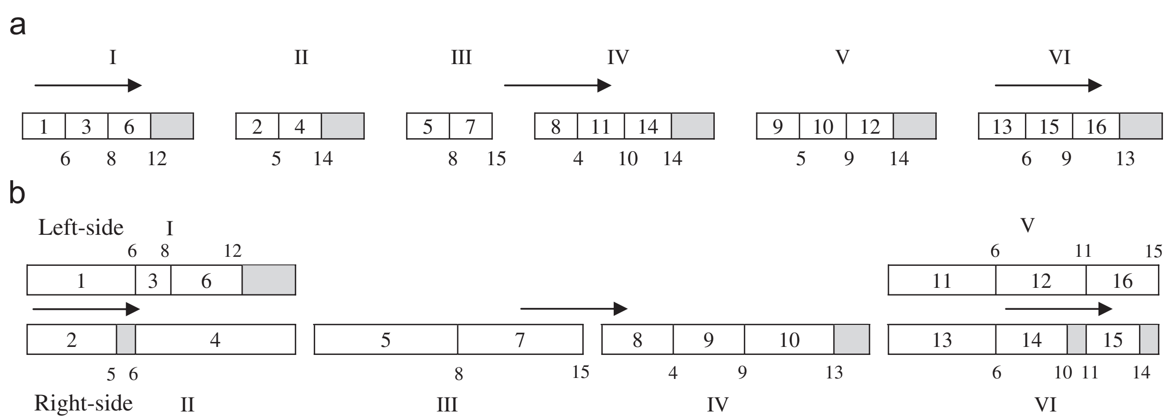

Fig. 2. The task assignments of (a) a one-sided assembly line and (b) a two-sided assembly line with a cycle time of 15.

high-volume products, such as trucks and buses [7]. In two-sided assembly lines, some tasks can be assigned to only one side of the two sides: L (left) and R (right)-type tasks, while others can be assigned to either side of the line: E (either)-type tasks. An example problem from Lee et al. [7] is used to clarify the two-sided assembly line balancing concepts. Precedence relations among tasks, task times, and preferred operation directions of tasks of the example problem are given in Fig. 1. The number of tasks is 16.

In Fig. 1, the numbers in the nodes represent the tasks, the labels (t_i, d_i) below the nodes represent the completion time of task i, and the preferred operation direction (L, R, E) of task i, respectively. L indicates that the task i should be assigned to a left-side station and R indicates that the task i should be assigned to a right-side station. Finally, E indicates that the task i can be performed at either side of the line. The directed arrow between node h and node i implies that task h immediately precedes task i. Fig. 2 illustrates the optimal solution obtained from a one-sided assembly line and a two-sided assembly line assuming a cycle time of 15 time units. The preferred operation directions of tasks assumed to be R only in one-sided assembly line balance.

In Fig. 2, task numbers are placed at their relevant positions inside the bars. For every task, its starting time and its finishing time are shown alongside the bars. Shaded rectangles indicate either unavoidable delay between two consecutive tasks, or idle time at the end of the cycle time. The one-sided assembly line balance given in Fig. 2(a) has six stations. Operators can perform tasks continuously without any interruption since the idle time of a station is concentrated at the end of the cycle time. The two-sided assembly line balance given in Fig. 2(b) has also six stations. A pair of two directly facing stations called a mated-station (e.g. stations I and II) and one of them calls the other as a companion. In the solution, the number of mated-stations (i.e., line length) is four. While balancing two-sided assembly lines, idle time is sometimes unavoidable even between tasks assigned to the same station. Suppose that task h is assigned to a station and task i is assigned to the companion of this station. Task i cannot be started unless task h completed. Therefore, the sequence-depended finishing time of tasks are taken into account, unlike a one-sided assembly line. In Fig. 2(b), for example, task 1 is assigned to the left-side of mated-station and tasks 2, and 4 are assigned to the right-side of the same mated-station. Task 4 cannot be started at the finishing time of task 2, since task 1 is the immediate predecessor of task 4. So the starting time of task 4 is equal to the finishing time of task 1.

According to Bartholdi [8], in practice, a two-sided assembly line can provide several advantages over a one-sided assembly line. These are: (i) the assembly line length can be shorter than a one-sided assembly line; (ii) it can reduce material handling cost, workers movement, set up time, and the amount of throughput time; and (iii) it can also reduce cost of tools, and fixtures.

In the mathematical complexity, one-sided assembly line balancing problem (OALBP) is NP-hard class of combinatorial optimization problems [9]. The combinatorial structure of this problem makes it difficult to obtain an optimal solution when the problem size increases. Two-sided assembly line balancing problem (TALBP) is also NP-hard class. In addition to the mathematical complexity of OALBP, TALBP has also an additional level of complexity, since the tasks have restrictions on the operation direction [8].

The TALBP can be classified as [7,10] TALBP-I: minimization of the number of mated-stations (i.e., the line length) for a given cycle time, and TALBP-II: minimization of the cycle time for a given number of mated-stations. However, in TALBP-I, assume that there are two different solutions with the same number of mated-stations, where one of these solutions may be better balanced than the other one, since one of them may have fewer stations than the other. So, the number of stations should be considered as well as the number of mated-stations.

Table 1
Summary of the literature on two-sided assembly lines.

Authors	Methodology	Type of problem
Bartholdi [8]	An interactive program with an assignment rule	TALBP-I
Kim et al. [11]	Genetic algorithm	TALBP-I (with positional constraints)
Lee et al. [7]	Group assignment procedure	TALBP-I and TALBP-II
Baykasoglu and Dereli [12]	Ant colony optimization algorithm	TALBP-I (with zoning restrictions)
Hu et al. [13]	Station-oriented enumerative algorithm	TALBP-I
Simaria and Vilarinho [14]	Mixed integer programming formulation and ant colony optimization algorithm	TALBP-I (with zoning restrictions)
Kim et al. [15]	Mixed integer programming formulation and genetic algorithm	TALBP-II
Wu et al. [10]	Branch-and-bound algorithm	TALBP-I

Although many research is done for OALBP, TALBP is studied by considerably few researchers. TALBP was first addressed by Bartholdi [8]. He designed an interactive program that assists assembly line managers to assign tasks and presented an assignment rule. Kim et al. [11] presented a genetic algorithm approach to solve the problem. Lee et al. [7] developed an assignment procedure for TALBP with two decision criteria, such as work relatedness and work slackness. Baykasoglu and Dereli [12] developed an ant colony optimization based heuristic algorithm to solve TALBP with zoning constraints. Hu et al. [13] presented a station-oriented enumerative algorithm for TALBP. Simaria and Vilarinho [14] addressed two-sided mixed-model assembly line balancing problem. They developed a mathematical programming model which is only used as a means to formally describe the problem, and presented an ant colony optimization algorithm to solve the problem. The mathematical programming model proposed by Simaria and Vilarinho [14] has high complexity. So, optimal solutions cannot be obtained by using this model. Kim et al. [15] presented a mathematical formulation and a genetic algorithm approach for TALBP with the objective of minimizing the cycle time for a given number of mated-stations. Wu et al. [10] presented a formal formulation of TALBP-I problem, and they developed a branch-and-bound algorithm to solve the problem optimally. Summary of the studies conducted on two-sided assembly lines is also given in Table 1.

As it can be seen in Table 1, three exact solution methodologies are developed to solve TALBP optimally [10,14,15]. In this paper, a new mixed integer programming (MIP) model is presented to solve TALBP-I with single-model production environment, and deterministic task completion times. The proposed model is based on the mathematical model proposed by Kim et al. [15]. The mathematical model of Kim et al. [15] is modified for solving TALBP-I optimally.

In many of the solution techniques for ALBP, only one objective was considered, such as minimizing the number of stations for a given cycle time, minimizing the cycle time for a given number of stations, etc. However, in real life applications, it may be necessary to consider more than one objective simultaneously. When the target values of the objectives can be easily determined precisely by the decision maker(s), goal programming (GP) introduced by Charnes and Cooper [16] is useful. Few studies using GP approach to ALBP have been reported in the literature [17–23]. On the other hand, it may be difficult for the decision maker(s) to precisely determine the target value of each objective, since the target values may be imprecise, vague, or uncertain. Fuzzy set theory is one of the useful tools for dealing with imprecision [24]. According to fuzzy set theory, imprecise, vague, or uncertain goals or constraints can be determined with triangular, trapezoidal, or linear membership functions [25]. There are only two studies using the fuzzy goal programming model (FGP) to ALBP reported in the literature [26,27]. For especially large-sized problems, many researchers studied to develop modern heuristic approaches to solve multi-objective ALBPs [28–30]. All multiple-criteria decision-making (MCDM) researches for ALBP are concerned with the straight or U-type ALBP.

To the best of knowledge of the authors, there is no published study dealing with MCDM aspects of TALBP in the literature. In this study, an MIP model is presented for the TALBP-I, firstly. Then, a mixed integer goal programming model (MIGP) is developed for the TALBP with precise and certain objectives. And also, a fuzzy mixed integer goal programming (FMIGP) model is developed for the TALBP with imprecise, vague, or uncertain goal value of each objective. The proposed models, MIGP and FMIGP, are the first MCDM approaches to the TALBP.

In TALBP, the primary objective is either to minimize the number of mated-stations/the number of stations or minimize the cycle time. Lee et al. [7] and Kim et al. [15] noted that various meaningful objectives, e.g. minimizing the number of mated-stations, minimizing the cycle time, minimizing workload deviation, maximizing work relatedness, maximizing work slackness and multiple objectives, may be considered in a real TALBP. Decision maker(s) may desire to develop a model considering the specific characteristics of the problem with multiple objectives. In this study, the number of mated-stations, cycle time and the number of tasks assigned per station goals which are most frequently used in the literature are considered to design our MIGP and FMIGP models.

The reminder of this paper is organized as follows. After this introduction, the MIP formulation of TALBP-I is presented. In Section 3, the GP in solving multi-objective problems is considered, and MIGP model for TALBP with precise goals is developed. The FGP is considered, and FMIGP model for the TALBP with multiple objectives is formulated in Section 4. In Section 5 an example problem is solved using the proposed MIGP and FMIGP models. In Section 6 a set of small-sized problems taken from literature are solved optimally using the proposed MIP model, and a test problem taken from literature is used to illustrate the effectiveness and efficiency of the proposed MIGP and FMIGP models. Finally, some conclusions and future research directions are presented in the final section. The notations used in the models are listed in Appendix A.

2. Mathematical formulation of TALBP-I

Kim et al. [15] developed a mathematical model for the TALBP-II. This model is the only published mathematical model for the TALBP-II in the literature. The primary objective of the model is minimizing the cycle time for a given number of mated-stations. The mathematical model formulation of TALBP-II is as follows [15]:

$$\text{Minimize} \quad \text{ct} \tag{1}$$

subject to

$$\sum_{j \in J} \sum_{k \in K(i)} x_{ijk} = 1 \quad \forall i \in I \tag{2}$$

$$\sum_{g \in J} \sum_{k \in K(h)} g \cdot x_{hgk} - \sum_{j \in J} \sum_{k \in K(i)} j \cdot x_{ijk} \leqslant 0 \quad \forall i \in I - P_0,\ \ h \in P(i) \tag{3}$$

$$t_i^f \leqslant ct \quad \forall i \in I \tag{4}$$

$$t_i^f - t_h^f + \psi \cdot \left(1 - \sum_{k \in K(h)} x_{hjk}\right) + \psi \cdot \left(1 - \sum_{k \in K(i)} x_{ijk}\right) \geqslant t_i \quad \forall i \in I - P_0,\ \ h \in P(i), j \in J \tag{5}$$

$$t_p^f - t_i^f + \psi \cdot (1 - x_{pjk}) + \psi \cdot (1 - x_{ijk}) + \psi \cdot (1 - z_{ip}) \geqslant t_p \quad \forall i \in I, \; p \in \{r | r \in I - (P_a(i) \cup S_a(i) \cup C(i)) \text{ and } i < r\},\ \ j \in J, k \in K(i) \cap K(p) \tag{6}$$

$$t_i^f - t_p^f + \psi \cdot (1 - x_{pjk}) + \psi \cdot (1 - x_{ijk}) + \psi \cdot z_{ip} \geqslant t_i \quad \forall i \in I, \; p \in \{r | r \in I - (P_a(i) \cup S_a(i) \cup C(i)) \text{ and } i < r\},\ \ j \in J, k \in K(i) \cap K(p) \tag{7}$$

$$x_{ijk} \in \{0,1\} \quad \forall i \in I, j \in J, k \in K(i) \tag{8}$$

$$z_{ip} \in \{0,1\} \quad \forall i \in I, p \in \{r | r \in I - (P_a(i) \cup S_a(i) \cup C(i)) \text{ and } i < r\} \tag{9}$$

$$t_i^f \geqslant t_i, \quad \forall i \in I \tag{10}$$

Constraint (2) is the assignment constraint and it ensures that each task is assigned to exactly one station. Constraint (3) is the precedence constraint and it ensures that all precedence relations among tasks are satisfied. Constraint (4) is the cycle time constraint and it ensures that each of the finishing time of tasks does not exceed the cycle time. Constraints (5)–(7) control the sequence-dependent finishing time of tasks. For every pair of task i and task h, if task h is an immediate predecessor of task i, and they are assigned to the same mated-station j, then constraint (5) becomes active, i.e., $t_i^f - t_h^f \geqslant t_i$. If two tasks do not have any precedence relations, and they are assigned to the same station (j,k), then constraints (6) and (7) become active. If task i is assigned to an earlier station than task p, constraint (6) becomes $t_p^f - t_i^f \geqslant t_p$. Otherwise, constraint (7) becomes $t_i^f - t_p^f \geqslant t_i$. Constraints (8) and (9) are the integrality constraints.

The primary objective of the proposed model is the minimization of the number of mated-stations (i.e., the line length) for a given cycle time. As mentioned before, a solution may be better balanced than the other one in two different obtained solutions, since one of them may have fewer stations than the other. Therefore, the number of stations should also be minimized as a secondary objective. Wu et al. [10] addressed this kind of problem with a secondary objective of minimizing the number of required stations. They presented the formal formulation of the problem only. In this study, an MIP model is developed for TALBP-I to solve optimally. The proposed mathematical model formulation of TALBP-I is as follows:

$$\text{Minimize} \quad \sum_{j \in J} (F_j + G_j) + \varepsilon \cdot \sum_{j \in J} \sum_{k=1,2} U_{jk} \tag{11}$$

subject to

Constraints (2)–(10), and

$$\sum_{i \in I} x_{ijk} - \|W_{jk}\| \cdot U_{jk} \leqslant 0 \quad \forall j \in J, k \in K(i) \tag{12}$$

$$\sum_{k=1,2} U_{jk} - 2 \cdot F_j - G_j = 0 \quad \forall j \in J \tag{13}$$

$$U_{jk} \in \{0,1\} \quad \forall j \in J, k = 1,2 \tag{14}$$

$$F_j, G_j \in \{0,1\} \quad \forall j \in J \tag{15}$$

Objective function (11) minimizes the number of mated-station (i.e., the line length) as the primary objective, and it also minimizes the number of stations (i.e., the number of operator) as a secondary objective. Constraint (12) is the station constraint developed by Deckro [18] for straight OALBP. It is modified for TALBP. If station (j,k) is utilized, i.e., if a task is assigned to station (j,k), then U_{jk} will be equal one; otherwise U_{jk} will be equal zero. Constraint (13) ensures that if the left-side station of a mated-station $(j,1)$, and the right-side station of the same mated-station $(j,2)$ are utilized together, then F_j will be equal to one, and G_j will be equal to zero; otherwise G_j will be equal to one, and F_j will be equal to zero. Therefore, total number of mated-stations is equal to the sum of F_j and G_j for all j. Constraints (14) and (15) are the integrality constraints.

Some tasks are forced to be assigned together to the same station (e.g. tasks may require the same tool, or fixture, and they may be performed on the same station using one tool, or fixture in order to reduce the cost of tools and fixtures), and other tasks are prohibited from being assigned to the same station (e.g. some tasks must not be assigned to the same station due to the their specific applications, such as welding and painting tasks). These constraints are known as compatible (positive) and incompatible (negative) zoning constraints, respectively. The following constraints (16) and (17) represent compatible and incompatible zoning constraints, respectively. Constraint (16) ensures that compatible tasks h and i are assigned to the same station (j,k), and constraint (17) ensures that incompatible tasks h and i are not assigned to the same station (j,k). In the equations, CZ and IZ represent sets of pairs of tasks for compatible and incompatible zoning, respectively.

$$x_{hjk} - x_{ijk} = 0 \quad \forall (i,h) \in CZ,\ \ j \in J, k \in K(i) \cap K(h) \tag{16}$$

$$x_{hjk} + x_{ijk} \leqslant 1 \quad \forall (i,h) \in IZ,\ \ j \in J, k \in K(i) \cap K(h) \tag{17}$$

3. GP formulation

3.1. Goal programming

GP is a branch of MCDM and is perhaps the oldest MCDM technique, and it is widely used to solve multi-objective problems. GP was first introduced by Charnes and Cooper [16]. Tamiz et al. [31] reviewed the current state-of-the-art in GP. The general aim of GP is the optimization of several conflicting goals precisely defined by the decision maker(s) by minimizing the deviations from the target values. The original objectives are expressed as a linear equation with target values and two auxiliary variables. This two auxiliary variables represent under-achievement of the target value by negative deviation (d^-) and over-achievement of the target value by positive deviation (d^+). The unwanted deviations between target values of objectives are minimized hierarchically. Hence, the goals of primary importance are satisfied first, and it is only then the goals of second importance are considered, and so forth. This variant of GP used in this paper is known as lexicographic GP (LGP), also known as non-Archimedean or pre-emptive GP. The framework of LGP model is formulated as follows [32]:

$$LEXMIN\ a = \left[\sum_{q \in pr_1} (\alpha_q \cdot d_q^- + \beta_q \cdot d_q^+), \ldots, \sum_{q \in pr_r} (\alpha_q \cdot d_q^- + \beta_q \cdot d_q^+), \ldots, \sum_{q \in pr_Q} (\alpha_q \cdot d_q^- + \beta_q \cdot d_q^+) \right] \tag{18}$$

subject to

$$f_s(x) =, \leqslant, \text{or} \geqslant b_s \quad s \in S \tag{19}$$

$$g_q(x) + d_q^- - d_q^+ = tv_q \quad q \in pr_r,\ \ r \in Q \tag{20}$$

$$d_q^-, d_q^+ \geqslant 0 \quad q \in pr_r \tag{21}$$

where pr_r represents the index set of goals placed in the rth priority level, and α_q and β_q are the weighting factors for d_q^+ and d_q^-, respectively. $f_s(x)$ is the system constraint, $g_k(x)$ is the goal constraint and tv_q is the target value of goal q.

3.2. Proposed MIGP model

The aim of the proposed MIGP model is to find an optimal two-sided assembly line balance that minimizes the number of mated-station, cycle time and the task load of stations hierarchically. These goals are the mostly used goals in the literature [22,23,26,27]. In addition to these goals, several goals can also be considered [17].

Goal 1: Number of mated-stations utilized in two-sided assembly line should not exceed *IST*.

If a specified number of mated-stations is imposed by the decision maker(s), then the following constraint can be written:

$$\sum_{j\in J}(F_j + G_j) \leqslant IST \tag{22}$$

The goal constraint of the number of mated-stations with deviational variables is as follows:

$$\sum_{j\in J}(F_j + G_j) + d_1^- - d_1^+ = IST \tag{23}$$

The minimization of d_1^+ to zero ensures that the total number of mated-stations will be *IST* or less.

Goal 2: Finish time of a task should not exceed *C*.

The goal constraint of the cycle time given in constraint (4) with deviational variables is as follows:

$$t_i^f + d_{2i}^- - d_{2i}^+ = C \quad \forall i \in I \tag{24}$$

The minimization of the sum of d_{2i}^+ to zero ensures that the cycle time of the system will be *C* or less.

Goal 3: Number of tasks per stations should not exceed *TSK*.

If a specified total number of tasks which are assigned to each station is imposed by the decision maker(s) due to the some physical, or technological reasons, then the following constraint can be written:

$$\sum_{i\in I} x_{ijk} \leqslant TSK \quad \forall j \in J,\ \ k \in K(i) \tag{25}$$

The goal constraint of the task load with deviational variables is as follows:

$$\sum_{i\in I} x_{ijk} + d_{3jk}^- - d_{3jk}^+ = TSK \quad \forall j \in J,\ \ k \in K(i) \tag{26}$$

The minimization of the sum of d_{3jk}^+ to zero ensures that the task count at each station will be *TSK* or less.

In LGP, it is assumed that the decision maker(s) can explicitly define all the target value of the goals. Determining the target value of the goals is a difficult task for the decision maker(s). The decision maker(s) should determine the target value of the goals considering the specific conditions of the problem. No calculation is necessary to get the values of *IST*, *C* and *TSK*. These values are completely determined by the decision maker(s) considering the specific situations of the problem.

Bartholdi [8] noted that a two-sided assembly line can be more space-efficient since the line length of a two-sided assembly line can be shorter than the line length of a one-sided assembly line. He also noted that a shorter line can provide some advantages such as lower material handling costs and lower tools and fixtures costs. Moreover, he mentioned that a two-sided assembly line can require fewer operators than a one-sided assembly line. Because of the specific characteristics of two-sided assembly lines, the line length has more importance than the number of operators. As shown in Fig. 2, both the one-sided assembly line balance and the two-sided assembly line balance have the same number of stations. However, two-sided assembly line needs a shorter physical line length than the one-sided assembly line. Therefore, in the MIP model, minimizing the number of mated-stations is used as the primary objective. However, as mentioned before, minimizing the number of stations should also taken into consideration if the cycle time is given.

The number of stations depends on the number of mated-stations. However, the number of stations does not clearly determined for a given number of mated-stations. Due to the operation direction constraints and the precedence constraints among tasks, sometimes tasks may be assigned to only one side of a mated-station. Therefore the number of stations may not be equal to 2 $*$ the number of mated-stations or 2 $*$ the number of mated-stations-1. As a result, it can be said that the number of stations is less than or equal to 2 $*$ the number of mated-stations (i.e., 1 $\leqslant$ the number of stations $\leqslant$ 2 $*$ the number of mated-stations). This can be seen at Fig. 2b clearly. This condition is true when a fixed cycle time is given. On the contrary, in the MIGP model, the minimization of the cycle time is considered as a goal. The target value of cycle time goal is determined by the decision maker(s) according to his/her preferences. Gökçen and Ağpak [23] and Toklu and Özcan [26] reported that the number of stations and cycle time goals are conflicting goals. When the cycle time decreases enough, the number of stations increases. And also, the maximum capacity of the line (i.e., stations) is used. As a result, it is not necessary to consider the number of stations as a goal, due to the number of mated-stations is already considered as a goal.

The proposed MIGP model for the TALBP with multiple precise objectives is formulated as follows:

$$LEXMIN \left\{ d_1^+, \sum_{i\in I} d_{2i}^+, \sum_{j\in J} \sum_{k=1,2} d_{3jk}^+ \right\} \tag{27}$$

subject to

Goal constraints: (23), (24) and (26),

System constraints: (2), (3), (5)–(10), (12)–(15) and

Non-negativity constraints:

$$d_1^-, d_1^+ \geqslant 0 \tag{28}$$

$$d_{2i}^-, d_{2i}^+ \geqslant 0 \quad i \in I \tag{29}$$

$$d_{3jk}^-, d_{3jk}^+ \geqslant 0 \quad \forall j \in J,\ \ k = 1,2 \tag{30}$$

The positive and negative zoning constraints can also be added as precise goals by adding deviation variables. The goal constraints of the positive and negative zoning constraints given in constraints (16) and (17) are as follows, respectively:

$$x_{hjk} - x_{ijk} + d_{4ihjk}^- - d_{4ihjk}^+ = 0 \quad \forall (i,h) \in CZ,\ \ j \in J, k \in K(i) \cap K(h) \tag{31}$$

$$x_{hjk} + x_{ijk} + d_{5ihjk}^- - d_{5ihjk}^+ = 1 \quad \forall (i,h) \in IZ,\ \ j \in J, k \in K(i) \cap K(h) \tag{32}$$

The minimization of the sum of the deviational variables d_{4ihjk}^-, d_{4ihjk}^+ to zero ensures that the compatible tasks h and i will be assigned to the same station for $\forall(i,h) \in CZ$. Similarly, the minimization of the sum of the deviational variables d_{5ihjk}^+ to zero ensures that the incompatible tasks h and i will not be assigned to the same station for $\forall(i,h) \in IZ$.

4. FGP formulation

4.1. Fuzzy goal programming

FGP using the membership functions was initially described by Narasimhan [33], and many studies of FGP have been presented in the literature. Chanas and Kuchta [34] provide an extensive literature survey and classification to FGP models.

A FGP problem containing Q fuzzy goals ($Z_q(x)$) with solution set of x is defined as follows:

$$\text{Optimize} \quad Z_q(x) \succ g_q (\text{or } Z_q(x) \prec g_q) \quad q \in Q \tag{33}$$

$$Ax \leqslant b \tag{34}$$

$$x \geqslant 0 \tag{35}$$

where $Z_q(x) \succ g_q$ (or$Z_q(x) \prec g_q$) indicates that the qth fuzzy goal is approximately greater than or equal to (approximately less than or equal to) the aspiration level g_q. The linear membership function μ_q for the qth fuzzy goal is defined as [35];

$$\mu_q = \begin{cases} 1 & \text{if } Z_q(x) \geqslant g_q \\ \dfrac{Z_q(x) - l_q}{g_q - l_q} & \text{if } l_q < Z_q(x) < g_q \\ 0 & \text{if } Z_q \leqslant l_q \end{cases} \tag{36}$$

for $Z_q(x) \succ g_q$ and as

$$\mu_q = \begin{cases} 1 & \text{if } Z_q(x) \leqslant g_q \\ \dfrac{u_q - Z_q(x)}{u_q - g_q} & \text{if } g_q < Z_q(x) < u_q \\ 0 & \text{if } Z_q \geqslant u_q \end{cases} \tag{37}$$

for $Z_q(x) \prec g_q$, where l_q (or u_q) is the lower (or upper) tolerance limit for the qth fuzzy goal $Z_q(x) \succ g_q$ (or$Z_q(x) \prec g_q$).

Tiwari et al. [36] proposed a weighted additive model to solve this problem which uses flexibility to determine the priority of the fuzzy goals. The model is defined as follows:

$$\text{Maximize} \quad \sum_{q=1}^{Q} w_q \mu_q \tag{38}$$

$$\mu_q \leqslant \frac{Z_q(x) - l_q}{g_q - l_q} \left(\text{or } \mu_q \leqslant \frac{u_q - Z_q(x)}{u_q - g_q} \right) \quad q \in Q \tag{39}$$

$$Ax \leqslant b \tag{40}$$

$$x, \mu_q \geqslant 0 \quad q \in Q \tag{41}$$

$$\mu_q \leqslant 1 \quad q \in Q \tag{42}$$

where w_q is the weight of the qth fuzzy goal constraint and μ_q is the achievement degree of the qth fuzzy goal:

$$\sum_{q=1}^{Q} w_q = 1 \quad q \in Q \tag{43}$$

4.2. Proposed FMIGP model

In this study, the goals which are given at Section 3 are considered as the fuzzy environment.

The first fuzzy goal is the number of mated-stations in the two-sided assembly line that is approximately less than or equal to IST:

$$Z_1 = \sum_{j \in J} (F_j + G_j) \prec IST \tag{44}$$

The second fuzzy goal is the finishing time of a task is approximately less than or equal to C:

$$Z_2 = t_i^f \prec C \quad \forall i \in I \tag{45}$$

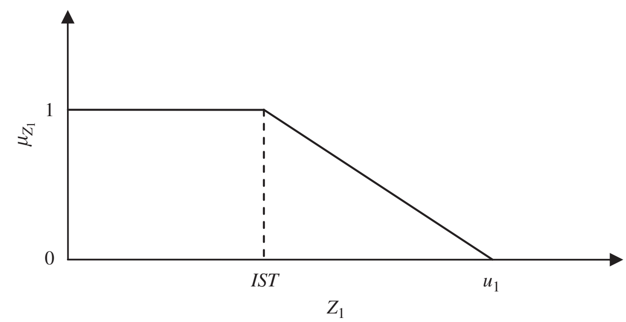

Fig. 3. Membership function of the fuzzy goal representing the number of mated-stations.

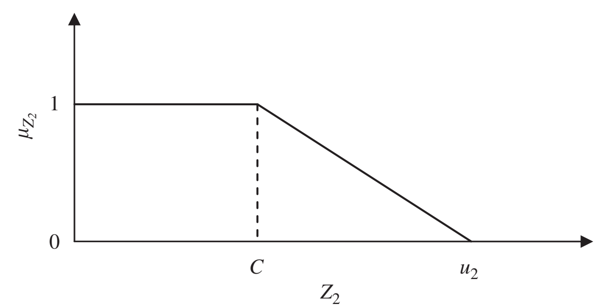

Fig. 4. Membership function of the fuzzy goal representing the cycle time.

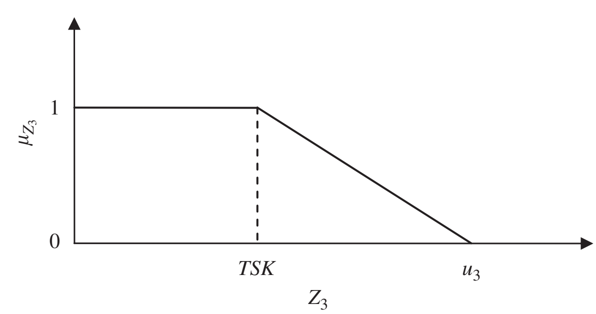

Fig. 5. Membership function of the fuzzy goal representing the task load of stations.

The third fuzzy goal is the total number of tasks which are assigned to each station is approximately less than or equal to TSK:

$$Z_3 = \sum_{i \in I} x_{ijk} \prec TSK \quad \forall j \in J, \ k \in K(i) \tag{46}$$

Assume that u_1, u_2 and u_3 are the upper tolerance limit and IST, C and TSK are the lower tolerance limit of the fuzzy goals Z_1, Z_2 and Z_3imposed by the decision maker(s), respectively. The linear membership function μ_{Z_q} for the qth fuzzy goal is defined as follows (Figs. 3–5):

$$\mu_{Z_1} = \begin{cases} 1 & \text{if } Z_1 \leqslant IST \\ (u_1 - Z_1)/(u_1 - IST) & \text{if } IST < Z_1 < u_1 \\ 0 & \text{if } Z_1 \geqslant u_1 \end{cases} \tag{47}$$

$$\mu_{Z_2} = \begin{cases} 1 & \text{if } Z_2 \leqslant C \\ (u_2 - Z_2)/(u_2 - C) & \text{if } C < Z_2 < u_2 \\ 0 & \text{if } Z_2 \geqslant u_2 \end{cases} \tag{48}$$

$$\mu_{Z_3} = \begin{cases} 1 & \text{if } Z_3 \leqslant TSK \\ (u_3 - Z_3)/(u_3 - TSK) & \text{if } TSK < Z_3 < u_3 \\ 0 & \text{if } Z_3 \geqslant u_3 \end{cases} \tag{49}$$

The fuzzy goals are then converted to the following formulations:

$$\mu_{Z_1} \leqslant (u_1 - Z_1)/(u_1 - IST) \tag{50}$$

$$\mu_{Z_2} \leqslant (u_2 - Z_2)/(u_2 - C) \tag{51}$$

$$\mu_{Z_3} \leqslant (u_3 - Z_3)/(u_3 - TSK) \tag{52}$$

$$0 \leqslant \mu_{Z_1}, \mu_{Z_2}, \mu_{Z_3} \leqslant 1 \tag{53}$$

Finally, the proposed FMIGP model for TALBP with multiple imprecise objectives is formulated as follows:

$$\text{Maximize} \quad f(\mu) = w_1 \cdot \mu_{Z_1} + w_2 \cdot \mu_{Z_2} + w_3 \cdot \mu_{Z_3} \tag{54}$$

subject to

Fuzzy goal constraints:

$$(u_1 - \text{IST}) \cdot \mu_{Z_1} + \sum_{j \in J} (F_j + G_j) - u_1 \leqslant 0 \tag{55}$$

$$(u_2 - C) \cdot \mu_{Z_2} + t_i^f - u_2 \leqslant 0 \quad \forall i \in I \tag{56}$$

$$(u_3 - TSK) \cdot \mu_{Z_3} + \sum_{i \in I} x_{ijk} - u_3 \leqslant 0 \quad \forall j \in J, \;\; k \in K(i) \tag{57}$$

System constraints: (2), (3), (5)–(10), (12)–(15) and (53).

Objective function (54) is the fuzzy decision with the weighted achievement degrees of the fuzzy goals. In the proposed model, a set of desirable levels of the achievement degrees of the fuzzy goals can be determined as constraints, i.e., $\mu_{Z_q} \geqslant \delta_q$, where $\delta_q (0 \leqslant \delta_q \leqslant 1)$ is the desirable achievement degree for the qth fuzzy goal. The determination of a desirable achievement degree for a fuzzy goal is a difficult task for the decision maker(s). Some useful methods can be employed to represent a fuzzy goal's importance as a real number in the range of [0, 1] [37]. The positive and negative zoning constraints cannot be considered as fuzzy goals. Because a compatible zoning, or an incompatible zoning can be either satisfied or unsatisfied.

5. Illustrative example

The proposed MIGP and FMIFP models are illustrated using an example problem given in Fig. 1. Assume that the lower tolerance limits (*IST*, *C* and *TSK*) and the upper tolerance limits (u_1, u_2 and u_3) of the fuzzy goals Z_1, Z_2 and Z_3 are 4, 10, 2 and 6, 15, 4, respectively. The illustrative example is solved using GAMS (general algebraic modeling system) mathematical programming package for the following sequence of priorities of the goals: the number of mated-station goal (P_1) $\gg$ the cycle time goal (P_2) $\gg$ the task load goal (P_3). Hence, for the proposed MIGP model, the deviation variables to be minimized are prioritized as follows: first d_1^+, second d_{2i}^+ for all $i \in I$and third d_{3jk}^+ for all $j \in J, k{=}1,2$, and for the proposed FMIGP model, the weights of the achievement degrees of the fuzzy goals are fixed at $w_1 = 100/111$, $w_2 = 10/111$ and $w_3 = 1/111$ from high-priority to low-priority level. Only the lower tolerance limits of the goals are used in MIGP model. The solutions obtained with MIGP and FMIGP models are given in Table 2.

As shown in Table 2, in the results of MIGP, the cycle time goal is not satisfied but the number of mated-station goal and the task load goal are satisfied. The deviation of under-achievement of the number of mated-station goal (d_1^+) is zero, and the deviation of under-achievement of the task load goal (d_{3jk}^+) are equal to zero for all $j \in J, k = 1, 2$. The positive deviation variables (d_{2i}^+) of tasks 4, 5 and 8 are 5, 3 and 1, respectively; the remainders are zero. The FMIGP is also obtained similar results of MIGP. The achievement degrees of the fuzzy goals Z_1, Z_2 and Z_3 are 1.0, 0.0 and 1.0, respectively. The achievement degree of the number of stations goal is zero. MIGP model shows whether or not a goal is satisfied, but FMIGP model shows the satisfaction degree of the goals.

In the literature, various approaches have been developed to considering relative importance of fuzzy goals in FGP models [33,36–39]. In this paper, the weighted additive model proposed by Tiwari et al. [36] is used. The basic concept of the weighted additive model is to use a single linear weighted utility function to determine the overall preference of decision maker. The relative importance of goals is then determined by decision maker(s). In the weighted additive model of Tiwari et al. [36], heavy weights are given to higher priority level goals. Similar weights may cause the misleading results. For example, in illustrative example, suppose that the weights of the fuzzy goals determined by the decision maker are $w_1 = 0.5$, $w_2 = 0.4$and $w_3 = 0.1$ from high-priority to low-priority level. As a result, the achievement degrees of the fuzzy goals Z_1, $Z_2$and Z_3 are obtained to be 0.5, 0.8 and 1.0, respectively. In this case, the number of mated-station goal is not fully satisfied. But Z_1 has the highest priority level. In this study, the weight parameters are determined through preliminary experiments and they are fixed at 100/111, 10/111 and 1/111 from high-priority to low-priority level for all considered test problems. There are some approaches those have been developed to specify the relative weight of goals in the literature [40]. If the importance relations among the goals are not determined by the decision maker(s), then the FGP method of Aköz and Petrovic [41] can be used.

6. Computational experiments and results

6.1. Computational experiments

A set of small-sized problems (P9, P12, P16 and P24) are solved using the proposed mathematical model for TALBP-I. P9, P12 and P24 are taken from Kim et al. [11], and P16 is taken from Lee et al. [7]. In addition to this, the test problems are also solved under zoning constraints. The zoning constraints for the test problems P9, P12 and P24 are taken from Baykasoglu and Dereli [12]. For the test problem of P16, the compatible and incompatible zoning restrictions are generated as $\{3, 6, 7\}$and $\{8, 9, 10\}$, respectively. The proposed model is solved using the GAMS mathematical programming package on a Pentium IV 3.0 GHz PC with 512 MB RAM. The runs which are not finish interrupted at 7200 s. The results are shown in Table 3.

In Table 3, *NM* and *NS* represent the number of mated-stations and the number of stations, respectively. *LB* represents the lower bound of the number of stations presented by Hu et al. [13]. The calculation of *LB* of TALBP-I is as follows.

$$\text{Max} = \max\left\{\left\lceil \frac{LTotal}{ct} \right\rceil, \left\lceil \frac{RTotal}{ct} \right\rceil\right\} \tag{58}$$

$$LB = 2 \times \text{Max} + \max\left\{0, \left\lceil \frac{ETotal - (\text{Max} \times ct - LTotal) - (\text{Max} \times ct - RTotal)}{ct} \right\rceil\right\} \tag{59}$$

where, *LTotal*, *RTotal* and *ETotal* are the sum of completion time of L, R and E directional tasks, respectively.

6.2. Computational results

In this section, a small-sized test problem (P24) taken from Kim et al. [11] is used to illustrate the flexibility of the proposed MIGP and FMIGP models. The required data of the problem is given in Table 4.

In LGP, the target value for each objective has to be defined precisely. On the contrary, in FGP, the goals are determined in a fuzzy environment with the lower and upper tolerance limits of the goals. In this two MCDM technique, the priority of goals has to be given. Table 5 shows the lower tolerance limits (they are used as

Table 2
Summary of the results of the example problem for $P_1 \geqslant P_2 \geqslant P_3$.

Mated-station	MIGP		FMIGP	
	Assigned tasks (finish time)		Assigned tasks (finish time)	
	Left-side	Right-side	Left-side	Right-side
I	1[6], 4[15]	2[5], 5[13]	2[5], 4[15]	1[6], 5[14]
II	3[2], 6[6]	7[7], 8[11]	3[2], 6[6]	7[7], 10[11]
III	11[6], 14[10]	9[5], 10[9]	11[10], 14[14]	8[4], 9[9]
IV	12[5], 16[10]	13[6], 15[9]	12[5], 16[10]	13[6], 15[9]

Table 3
Results of MIP model.

Problem	Cycle time	Without zoning constraints				With zoning constraints		
			MIP			MIP		
			Optimal Solution			Optimal Solution		
		LB	NM	NS	CPU time (s)	NM	NS	CPU time (s)
P9	3	6	3	6	0.093	4	7	1.421
	4	5	3	5	0.296	3	5	0.234
	5	4	2	4	0.140	2	4	0.156
	6	3	2	3	0.109	2	3	0.203
P12	5	5	3	6	22.609	3	6	1.640
	6	5	3	5	12.042	3	5	4.656
	7	4	2	4	0.203	2	4	0.281
	8	4	2	4	1.125	2	4	2.562
P16	15	6	4	6	132.859	4	6	13.640
	16	6	3	6	2.031	4	6	30.234
	18	5	3	6	153.328	3	6	0.296
	19	5	3	5	18.125	3	6	0.359
	20	5	3	5	156.609	3	5	0.875
	21	4	3	5	399.640	3	5	1.046
	22	4	2	4	0.671	3	5	2.015
P24	18	8	4	8	<7200	4	8	<7200
	20	7	4	8	<7200	4	8	3999.265
	24	6	3	6	1621.437	3	6	6604.968
	25	6	3	6	<7200	3	6	<7200
	30	5	3	5	<7200	3	5	<7200
	35	4	2	4	259.671	2	4	242.890
	40	4	2	4	<7200	2	4	<7200

Table 4
Data of the problem.

Task	Immediate predecessor (s)	Task time	Preferred operation direction
1	–	3	L
2	–	7	L
3	–	7	R
4	–	5	R
5	2	4	L
6	2, 3	3	E
7	3	4	R
8	5	3	E
9	6	6	E
10	7	4	E
11	1	4	L
12	8, 9	3	L
13	9	3	E
14	9, 10	9	R
15	4	5	R
16	11	9	L
17	12	2	E
18	13	7	E
19	13, 14	9	E
20	15	9	R
21	16, 17	8	L
22	18	8	E
23	19, 20	9	R
24	20	9	E

Table 5
Lower and upper tolerance limits and the priority sequence of goals.

Run	The number of mated-station goal			The cycle time goal			The task load goal		
	IST	u_1	Priority level	*C*	u_2	Priority level	*TSK*	u_3	Priority level
1	3	5	P_1	20	25	P_2	4	7	P_3
2	3	5	P_2	20	25	P_3	4	7	P_1
3	3	5	P_3	20	25	P_1	4	7	P_2
4	4	6	P_1	15	20	P_3	2	5	P_2
5	4	6	P_2	15	20	P_1	2	5	P_3
6	4	6	P_3	15	20	P_2	2	5	P_1

the target value for each goal in MIGP only), and the upper tolerance limits with priority sequence of goals.

The values of the weights of the achievement degrees of the fuzzy goals are fixed at 100/111, 10/111, and 1/111 from high-priority level to low-priority level. All the test problems were solved using the GAMS mathematical programming package. The results are given in Table 6.

Table 6 shows the MIGP and FMIGP results. In Runs 1 and 4, due to the decision maker's preference, the number of mated-station is the most important criterion and the number of mated-station goal

Table 6
Results of MIGP and FMIGP models.

Run	MIGP			FMIGP		
	Goal conditions			Achievement degrees of fuzzy goals		
	Goal 1	*Goal* 2	*Goal* 3	μ_{Z_1}	μ_{Z_2}	μ_{Z_3}
1	S	U	U	1.000	0.200	0.667
2	U	S	S	0.500	1.000	1.000
3	U	S	S	0.500	1.000	1.000
4	S	U	U	1.000	0.000	0.667
5	U	S	S	0.000	1.000	1.000
6	U	S	S	0.000	1.000	1.000

S: satisfied.
U: unsatisfied.

is fully satisfied. In Runs 2 and 6, the task load goal which is the most important criterion due to the decision maker's preference is satisfied and the achievement degree of the task load goal is improved from 0.667 to 1.0 for each runs. Similarly, in Runs 3 and 5, the cycle time goal is fully satisfied.

The results of the MIGP model are similar to the results of the FMIGP model. In the MIGP model, the goals are considered one by one for a given priority level of the goals and the upper-level goals are optimized before the lower-level goals are considered. The MIGP model shows whether or not a goal is satisfied. However, the FMIGP model determines the achievement degrees of each goal and provides a high flexibility to the decision maker(s) for the determining the goals. Additionally, equal priorities between the goals can be evaluated in the FMIGP model using the same weight of the fuzzy goals (e.g. the weights can be fixed at $\frac{1}{3}$ for three fuzzy goals).

According to the results shown at Table 6, the number of mated-station goal conflicts with the cycle time goal, and also it conflicts with the tasks load goal. These results obtained with the proposed models are similar to those of GP model of Gökçen and Ağpak [23], and FGP model of Toklu and Özcan [26]. The computational times are high. All of the runs are interrupted after 7200 s. Hence, the computational times for large-sized problems such as P65 and P148 from Lee et al. [7] and P205 from Bartholdi [8] may be too high. Based on the decision maker's preference as shown in Table 6, the proposed MIGP and FMIGP models are capable of improving the value of goals.

7. Conclusions and future research directions

In real life applications, more than one conflicting objectives are considered simultaneously to obtain effective and realistic solutions. Some goals can be easily determined as precise goals by the decision maker(s). However, some goals should be determined as fuzzy goals, because these goals may be imprecise, vague, or uncertain. In this paper, a pre-emptive (lexicographic) MIGP model for precise goals, and an FMIGP model for imprecise goals are proposed to deal with the TALBP. The proposed MIGP and FMIGP models are the first MCDM approaches to this problem. They are capable of simultaneously optimizing more than one conflicting goals. Three objectives are considered. These are the minimization of the number of mated-stations, the minimization of the cycle time and the minimization of the number of tasks which are assigned to each station. An example problem is solved using MIGP and FMIGP models, and a computational study is conducted to illustrate the flexibility and the efficiency of the proposed MCDM approaches. The results of the run show the proposed models are valid, and they provide that the decision maker(s) can examine numerous scenarios regarding various conditions. Also, an MIP formulation is proposed to solve TALBP optimally. The proposed model minimizes the number of mated-stations as the primary objective, and it minimizes the number of stations as a secondary objective for a given cycle time. The zoning constraints are also considered in this model, and a set of small-sized problem taken from literature is solved using MIP model. Development of mathematical formulations based on the proposed models considering balancing mixed-model two-sided assembly lines should be of interest for further studies. And also, further developments on two-sided assembly lines should be made by considering different assembly line layouts, such as U-shaped assembly lines.

Appendix A. Notations used in model formulations

Indices

i, h, p, r	a task
j, g	a mated-station
k	a side of the line; $k = \begin{cases} 1 & \text{indicates a left-side station} \\ 2 & \text{indicates a right-side station} \end{cases}$
(j, k)	a station of mated-station j and its operation direction is k

Parameters

I	set of tasks; $I = \{1, 2, \ldots, i, \ldots, nt\}$
J	set of mated-stations; $J = \{1, 2, \ldots, j, \ldots, nms\}$
A_L	set of tasks which should be performed at a left-side station; $A_L \subset I$
A_R	set of tasks which should be performed at a right-side station; $A_R \subset I$
A_E	set of tasks which can be performed at either side of a station; $A_E \subset I$
$P(i)$	set of immediate predecessors of task i
$P_a(i)$	set of all predecessors of task i
$S(i)$	set of immediate successors of task i
$S_a(i)$	set of all successors of task i
P_0	set of tasks that have no immediate predecessors; $P_0 = \{i \in I \mid P(i) = \emptyset\}$
t_i	completion time of task i
ψ	a very large positive number
$C(i)$	set of tasks whose operation directions are opposite to operation direction of task i; $C(i) = \begin{cases} A_L & \text{if } i \in A_R \\ A_R & \text{if } i \in A_L \\ \emptyset & \text{if } i \in A_E \end{cases}$
$K(i)$	set of indicating the preferred operation directions of task i; $K(i) = \begin{cases} \{1\} & \text{if } i \in A_R \\ \{2\} & \text{if } i \in A_L \\ \{1, 2\} & \text{if } i \in A_E \end{cases}$

ct	cycle time
ε	a small positive value, $0<\varepsilon\leqslant 1/(2*nt+1)$
Wjk	subset of all tasks that can be assigned to station (j, k)
$\|Wjk\|$	number of tasks in subset Wjk
CZ	set of pairs of compatible tasks for positive zoning; $CZ=\{(i,h),\ldots,(p,r)\}$
IZ	set of pairs of incompatible tasks for negative zoning; $IZ=\{(i,h),\ldots,(p,r)\}$
IST	lower tolerance limit for the number of mated-stations
C	lower tolerance limit for cycle time
TSK	lower tolerance limit for the number of tasks which can be assigned to a station
u_1	upper tolerance limit for the number of mated-stations
u_2	upper tolerance limit for cycle time
u_3	upper tolerance limit for the number of tasks which can be assigned to a station
w_1	weight of the number of mated-station fuzzy goal
w_2	weight of the cycle time fuzzy goal
w_3	weight of the task load fuzzy goal

Decision variables

x_{ijk}	1, if task i is assigned to station (j, k); 0, otherwise
t_i^f	finish time of task i
F_j	1, if mated-station j is utilized for both sides of the line; 0, otherwise
G_j	1, if mated-station j is utilized for only side of the line; 0, otherwise
U_{jk}	1, if station (j, k) is utilized; 0, otherwise
μ_{Z_1}	achievement degree of the number of mated-station fuzzy goal
μ_{Z_2}	achievement degree of the cycle time fuzzy goal
μ_{Z_3}	achievement degree of the task load fuzzy goal

Indicator variables

z_{ip}	1, if task i is assigned earlier than task p in the same station; 0, if task p is assigned earlier than task i in the same station

Auxiliary variables

d_1^-	deviation of under-achievement of IST
d_1^+	deviation of over-achievement of IST
d_{2i}^-	deviation of under-achievement of C for task i
d_{2i}^+	deviation of over-achievement of C for task i
d_{3jk}^-	deviation of under-achievement of TSK for station (j, k)
d_{3jk}^+	deviation of over-achievement of TSK for station (j, k)
d_{4ihjk}^-	deviation of under-achievement of the positive zoning goal for tasks i, and h
d_{4ihjk}^+	deviation of over-achievement of the positive zoning goal for tasks i, and h
d_{5ihjk}^-	deviation of under-achievement of the negative zoning goal for tasks i, and h
d_{5ihjk}^+	deviation of over-achievement of the negative zoning goal for tasks i, and h

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