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# Combining contemporary and traditional project management tools to resolve a project scheduling problem

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ABSTRACT

In this paper we examine a construction project involving the building of large concrete slabs for three buildings in an office park complex. There are finish-to-start (FS) as well as start-to-start (SS) and finish-to-finish (FF) precedence relationships among the project activities. We prepare an initial project schedule using Microsoft Project and manually validate the results using the precedence diagramming method (PDM) procedure. When the client informs us that the schedule must be shortened we find that Microsoft Project does not have the capability for resolving our particular time/cost tradeoff issues. So we revert to the traditional approach for resolving time/cost tradeoffs in projects and develop an original linear programming formulation for the time/cost tradeoff problem when a project is modeled as a precedence diagram. By combining contemporary (Microsoft Project) and traditional (a linear programming time/cost tradeoff model) project management tools we are able to successfully resolve the scheduling issues associated with the slab construction project. Further, we demonstrate the anomalous effects of start-to-start (SS) and finish-to-finish (FF) relationships via our construction project example in which the solution to the time/cost tradeoff problem requires that certain activities be lengthened in order to shorten the project duration.



## 1. Introduction

Modern day project management can trace its roots to the development of PERT and CPM in the late 1950s and the creation of the Project Management Institute (PMI) in 1969. Since that time many new project management "tools" have been developed. Over the years several network modeling formats have been utilized to represent the precedence and sequencing among project activities. Originally, both the critical path method (CPM) [1] and the program evaluation and review technique (PERT) [2] utilized the event-on node (EON) format to construct project network models. The activity-on-node (AON) project network format was a component of a project scheduling method called MPM-Methode des Potentiels Metra, which was developed by Bernard Roy, a pioneering French OR analyst [3]. The introduction of the AON format in the US is attributed to John W. Fondahl of the Civil Engineering Department at Stanford University [4,5]. J. David Craig of the IBM Corporation is credited with developing the precedence diagramming method (PDM) as a project management application program for the IBM 1440 computer [6]. The PDM concept was later amplified and expanded by several authors including [6–8] and [9], among others.

The precedence diagramming method is an enhanced version of the AON format. It includes the capability to directly model "start-to-start" (SS), "finish-to-finish" (FF) and "start-to-finish" (SF) precedence relationships among activities as well as the traditional "finish-to-start" (FS) relationship. In addition, the precedence diagramming method can also incorporate lead-lag factors into the relationships. These concepts are illustrated in Exhibit 1.

Today, Microsoft Project (MSP) is arguably the most widely used project management software tool, at least on small-to-medium size projects. Based on a project activity list, MSP has the capability to "draw" project diagrams in the traditional AON format with finish-to-start precedence relationships (network view), display the project schedule in a calendar format (calendar view), and also accommodates precedence diagramming concepts and displays these relationships nicely in the Gantt Chart "view" it produces.

The time/cost tradeoff problem was introduced soon after the origination of the critical path method [10], and is one of the more frequently discussed topics in the project management literature. Notable examples of early contributors include: [11–16]. However, searches of the literature have failed to reveal any research dealing with time/cost tradeoffs when projects are represented in the precedence diagramming format.

In this paper we examine a construction project involving the building of concrete foundations (slabs) for three buildings in an

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**Precedence Diagramming**

The Precedence Diagramming Method (PDM) employs the Activity-On-Node (AON) format for representing project activities in a network model, and defines the following possible precedence relationships and related lead-lag factors among activities in the project network diagram.

```
1.)  Finish-to-Start (FS), A(i,j)

     [ Task (i) ]
                |<- A(i,j) ->[ Task (j) ]

2.)  Start-to-Start (SS), B(i,j)

     [ Task (i) ]
     |
     |<----- B(i,j) ->[ Task (j) ]

3.)  Finish-to-Finish (FF), C(i,j)

     [ Task (i) ]<--- C(i,j) --->|
                                 |
                      [ Task (j) ]

4.)  Start-to-Finish (SF), D(i,j)

     [ Task (i) ]
     |                  [ Task (j) ]
     |<---- D(i,j) -------------->|
```

**Exhibit 1**

office park complex. After the preparation of the project activity list, the data are entered into MSP and the initial project schedule is available for inspection. When the client informs the construction contractor that the schedule must be shortened, we find that MSP does not have the capability to help resolve our particular time/cost tradeoff issues. So we revert to the traditional linear programming approach for resolving time/cost tradeoffs in project network models. However, our search of the literature for a time/cost tradeoff model for a project network constructed in the precedence diagramming format fails to find such a formulation. Thus, to resolve the construction scheduling problem at hand, we develop an original linear programming formulation of the time/cost tradeoff problem for projects modeled using the precedence diagramming format. Interestingly, as pointed out by [17], the solution of the slab construction scheduling problem demonstrates the anomalous effects due to the combination of start-to-start (SS) and finish-to-finish (FF) precedence relationships among project activities, which sometimes requires that certain activities be lengthened (started earlier) in order to shorten the project duration.

## 2. Example problem

The following problem, based on an example from Wiest and Levy [8], has been modified to include data appropriate for time/cost tradeoff analysis. The problem involves a construction project, which requires the building of three adjacent concrete slabs. There are five activities associated with the construction of each slab for a total of 15 activities. Exhibit 2a is a manually-drawn diagram of the 15 project activities, which has been "marked-up" to include (and display) the precedence relationships among the project activities. Note that activities in Phase 1 of the project have precedence relationships with activities in both Phases 1 and 2 of the project, and that activities in Phase 2 have precedence relationships with activities in both Phases 2 and 3 of the project. Subsequently, the activity and precedence information depicted in the precedence diagram (Exhibit 2a) was transcribed into a project activity list (Exhibit 2b), which includes the normal time (in days) to perform the activities as well as summarizing the precedence relationships among the activities. Note that there is no column provided in the project activity list for "start-to-finish" (SF) relationships because there are none in this example, and—in fact—they occur very rarely in practice [7].

The data from the project activity list (Exhibit 2b) was entered into MSP, and the resulting Gantt Chart view is displayed in Exhibit 3a. The Gantt Chart view not only depicts the project schedule and critical path, but also illustrates the precedence relationships among activities in a meaningful way. Exhibit 3a provides the construction project manager with the following information: (a) if started on Monday, April 27th the project would be completed in 56 working days on Wednesday, July 11th, and (b) activities A1, A2, A3, A7, A8, A12, A13, A14 and A15 are critical under conventional assumptions. Note: While MSP has the capability to display AON network, calendar, and other "views" of the project, they will not be included in this presentation.

Further, analysis of the PDM project network reveals that the nine (9) critical activities actually form three (3) critical paths as illustrated in Exhibit 3b. (See Appendices A and B for an explanation of the PDM critical path analysis algorithm).

As can be seen in Exhibits 3a and 3b, PDM analysis also reveals extensions to conventional critical path concepts, which result from the presence of SS and FF precedence relationships. In conventional AON analysis employing only FS relationships, the critical path is a sequence of activities (usually with zero slack) connected by FS relationships. In this conventional perspective, the critical path progresses from the beginning of an activity through it to its completion and then to the beginning of the next activity along the path, e.g., (1)→(2). The precedence diagramming method extends critical path concepts in several ways. An SS relationship along the critical path indicates that the path precedes from the beginning of activity ($i$) to the beginning of

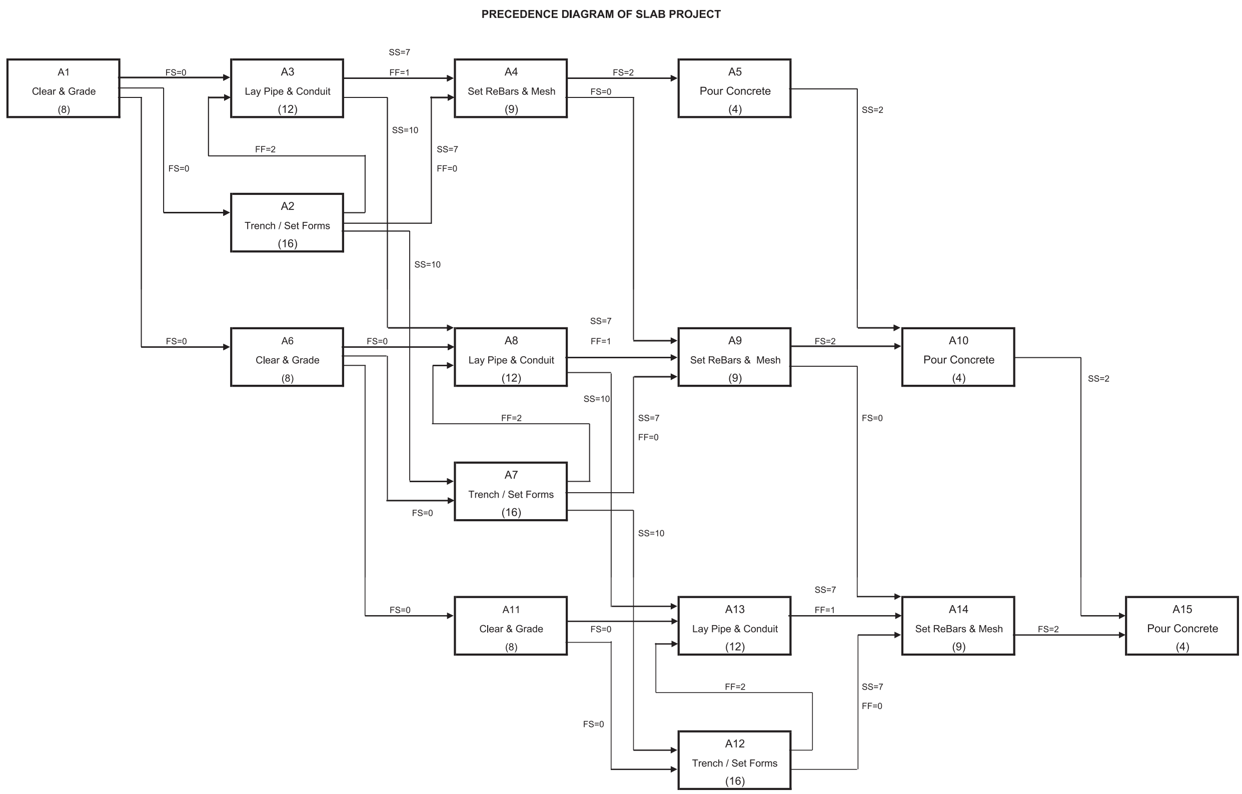



**Exhibit 2a**

**Construction Project Example / Activity List**

| Activity Number | Activity Description | Normal Duration N(j) | (FS) A(i,j) | (SS) B(i,j) | (FF) C(i,j) |
|---|---|---|---|---|---|
| | | | | | |
| A1 | Clear and Level Site (1) | 8 | | | |
| A2 | Dig Trench / Set Forms (1) | 16 | A1(0) | | |
| A3 | Lay Pipes & Conduit (1) | 12 | A1(0) | | A2(2) |
| A4 | Install Rebars & Mesh (1) | 9 | | A2(7)<br>A3(7) | A2(0)<br>A3(1) |
| A5 | Pour & Finish Concrete (1) | 4 | A4(2) | | |
| | | | | | |
| A6 | Clear and Level Site (2) | 8 | A1(0) | | |
| A7 | Dig Trench / Set Forms (2) | 16 | A6(0) | A2(10) | |
| A8 | Lay Pipes & Conduit (2) | 12 | A6(0) | A3(10) | A7(2) |
| A9 | Install Rebars & Mesh (2) | 9 | A4(0) | A7(7)<br>A8(7) | A7(0)<br>A8(1) |
| A10 | Pour & Finish Concrete (2) | 4 | A9(2) | A5(2) | |
| | | | | | |
| A11 | Clear and Level Site (3) | 8 | A6(0) | | |
| A12 | Dig Trench / Set Forms (3) | 16 | A11(0) | A7(10) | |
| A13 | Lay Pipes & Conduit (3) | 12 | A11(0) | A8(10) | A12(2) |
| A14 | Install Rebars & Mesh (3) | 9 | A9(0) | A12(7)<br>A13(7) | A12(0)<br>A13(1) |
| A15 | Pour & Finish Concrete (3) | 4 | A14(2) | A10(2) | |
| | | | | | |

**Activity details:**

- (1) Clear and level site
- (2) Dig foundation trench and set forms
- (3) Lay pipes for water, sewer & gas and conduit for electric, etc.
- (4) Install reinforcement rods (rebars) and mesh
- (5) Pour and finish concrete

**Exhibit 2b**

activity (k) with a lag of $B(i,k)$ time units, e.g., (3)→(8) and (8)→(13). An FF relationship along the critical path indicates that the path precedes from the completion of activity (i) to the completion of activity ($k$) with a lag of $C(i,k)$ time units, e.g., (2)→(3), (7)→(8) and (12)→(13). In addition, an FF relationship between activities ($i$,$j$) followed by an SS relationship between activities ($j$,$k$) causes activity ($j$) to be "reverse" critical, because the path precedes from the completion of ($j$) to the beginning of ($j$). For example, the sequence FF(2,3), SS(3,8) makes activity (3) "reverse" critical. When an activity is "reverse" critical an interesting "anomaly" occurs as illustrated and explained in Exhibit 4. In order to shorten the critical path, a "reverse" critical activity must be lengthened [17]. We will see this anomaly occur in the time/cost tradeoff analysis of our example problem.

Suppose at this point the client intercedes and states that the current project schedule is not acceptable and needs to be reworked—because the project must be completed by the end of the fiscal year (June 30, 2007). The subcontractor indicates that they

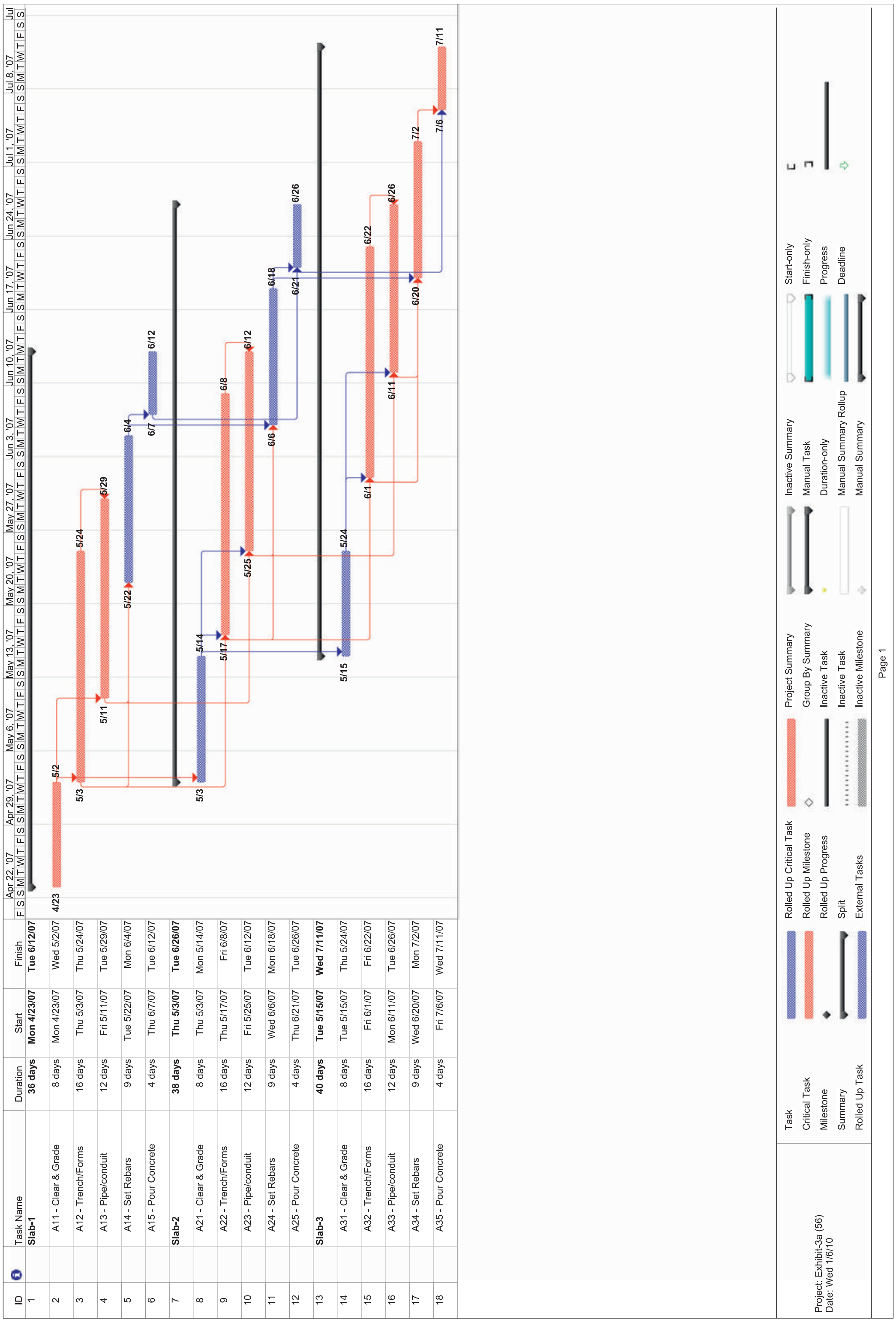

ID Task Name Duration Start Finish
1 Slab-1 36 days Mon 4/23/07 Tue 6/12/07
2 A11 - Clear & Grade 8 days Mon 4/23/07 Wed 5/2/07
3 A12 - Trench/Forms 16 days Thu 5/3/07 Thu 5/24/07
4 A13 - Pipe/conduit 12 days Fri 5/11/07 Tue 5/29/07
5 A14 - Set Rebars 9 days Tue 5/22/07 Mon 6/4/07
6 A15 - Pour Concrete 4 days Thu 6/7/07 Tue 6/12/07
7 Slab-2 38 days Thu 5/3/07 Tue 6/26/07
8 A21 - Clear & Grade 8 days Thu 5/3/07 Mon 5/14/07
9 A22 - Trench/Forms 16 days Thu 5/17/07 Fri 6/8/07
10 A23 - Pipe/conduit 12 days Fri 5/25/07 Tue 6/12/07
11 A24 - Set Rebars 9 days Wed 6/6/07 Mon 6/18/07
12 A25 - Pour Concrete 4 days Thu 6/21/07 Tue 6/26/07
13 Slab-3 40 days Tue 5/15/07 Wed 7/11/07
14 A31 - Clear & Grade 8 days Tue 5/15/07 Thu 5/24/07
15 A32 - Trench/Forms 16 days Fri 6/1/07 Fri 6/22/07
16 A33 - Pipe/conduit 12 days Mon 6/11/07 Tue 6/26/07
17 A34 - Set Rebars 9 days Wed 6/20/07 Mon 7/2/07
18 A35 - Pour Concrete 4 days Fri 7/6/07 Wed 7/11/07
Project: Exhibit-3a (56)
Date: Wed 1/6/10
Task
Critical Task
Milestone
Summary
Rolled Up Task
Rolled Up Critical Task
Rolled Up Milestone
Rolled Up Progress
Split
External Tasks
Project Summary
Group By Summary
Inactive Task
Inactive Task
Inactive Milestone
Inactive Summary
Manual Task
Duration-only
Manual Summary Rollup
Manual Summary
Start-only
Finish-only
Progress
Deadline
Page 1

Exhibit 3a

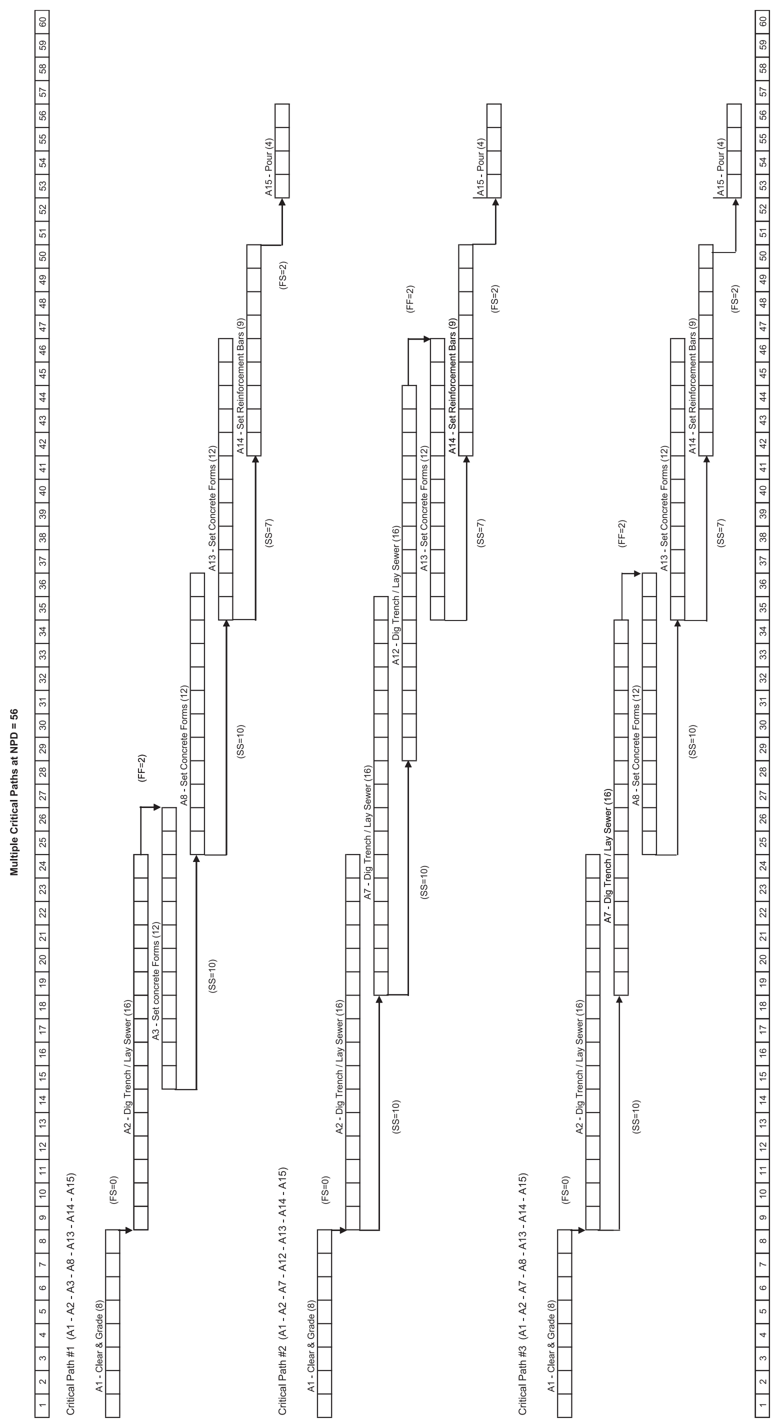
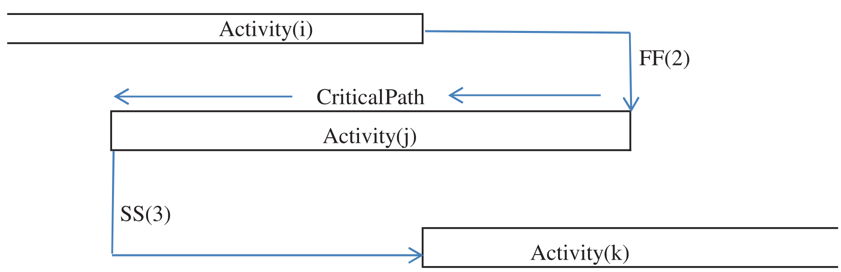

Multiple Critical Paths at NPD = 56
Critical Path #1 (A1 - A2 - A3 - A8 - A13 - A14 - A15)
A1 - Clear & Grade (8)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(FF=2)
A3 - Set concrete Forms (12)
(SS=10)
A8 - Set Concrete Forms (12)
(SS=10)
A13 - Set Concrete Forms (12)
(SS=7)
A14 - Set Reinforcement Bars (9)
(FS=2)
A15 - Pour (4)
Critical Path #2 (A1 - A2 - A7 - A12 - A13 - A14 - A15)
A1 - Clear & Grade (8)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(SS=10)
A7 - Dig Trench / Lay Sewer (16)
(SS=10)
A12 - Dig Trench / Lay Sewer (16)
(FF=2)
A13 - Set Concrete Forms (12)
(SS=7)
A14 - Set Reinforcement Bars (9)
(FS=2)
A15 - Pour (4)
Critical Path #3 (A1 - A2 - A7 - A8 - A13 - A14 - A15)
A1 - Clear & Grade (8)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(SS=10)
A7 - Dig Trench / Lay Sewer (16)
(FF=2)
A8 - Set Concrete Forms (12)
(SS=10)
A13 - Set Concrete Forms (12)
(SS=7)
A14 - Set Reinforcement Bars (9)
(FS=2)
A15 - Pour (4)

**Exhibit 3b**

Anomolous Effects of FF followed by SS Precedence Relationships




If Activities (i), (j) and (k) are on the critical path, the combination of an FF relationship between (i,j) and an SS relationship between (j,k) creates a situation where the critical path flows "backwards" thru Activity(j) causing it to be "reverse" critical. There are three options available to shorten the duration of the project; (a) shorten Activity(i), (b) shorten Activity(k), or (c) lengthen Activity(j). Lengthening Activity(j), i.e., starting it earlier, allows Activity(k) to start earlier, which shortens the project.

**Exhibit 4**

Example Problem / Activity List

| Activity | Description | Normal Time N(j) | Minimum Time M(j) | Cost/Period Z(j) |
|---|---|---|---|---|
| 1 | Clear 1 | 8 | 1 | 25 |
| 2 | Sewer 1 | 16 | 4 | 20 |
| 3 | Forms 1 | 12 | 2 | 15 |
| 4 | Rebars 1 | 9 | 1 | 10 |
| 5 | Pour 1 | 4 | 0 | 999 |
| 6 | Clear 2 | 8 | 1 | 25 |
| 7 | Sewer 2 | 16 | 4 | 20 |
| 8 | Forms 2 | 12 | 2 | 15 |
| 9 | Rebars 2 | 9 | 1 | 10 |
| 10 | Pour 2 | 4 | 0 | 999 |
| 11 | Clear 3 | 8 | 1 | 25 |
| 12 | Sewer 3 | 16 | 4 | 20 |
| 13 | Forms 3 | 12 | 2 | 15 |
| 14 | Rebars 3 | 9 | 1 | 10 |
| 15 | Pour 3 | 4 | 0 | 999 |

**Exhibit 5**

cannot start the project before Monday, April 23, 2007 because of prior commitments, cannot work on Saturdays, Sundays, or holidays because of contractual obligations, and also indicates that the planning should include one additional day for “site cleanup” and “other contingencies”. The subcontractor also provides cost estimates for changes in the durations of project activities (Exhibit 5).

At this point we look to Microsoft Project for help in rescheduling the project so that it is completed by June 28, 2007 (48 working days), leaving Friday, June 29, 2007 for site cleanup. Although Microsoft Project has a number of features that assist with resource scheduling and costing, it does not have a feature that will compress a project by $X$ days at minimum cost. So we revert to the traditional approach for resolving time/cost tradeoff problems in project management—a linear programming model.

## 3. Time/cost tradeoff analysis

An original time/cost tradeoff model for projects modeled in the precedence diagramming format is developed and presented in Appendices C1 and C2. The formulation for our slab construction

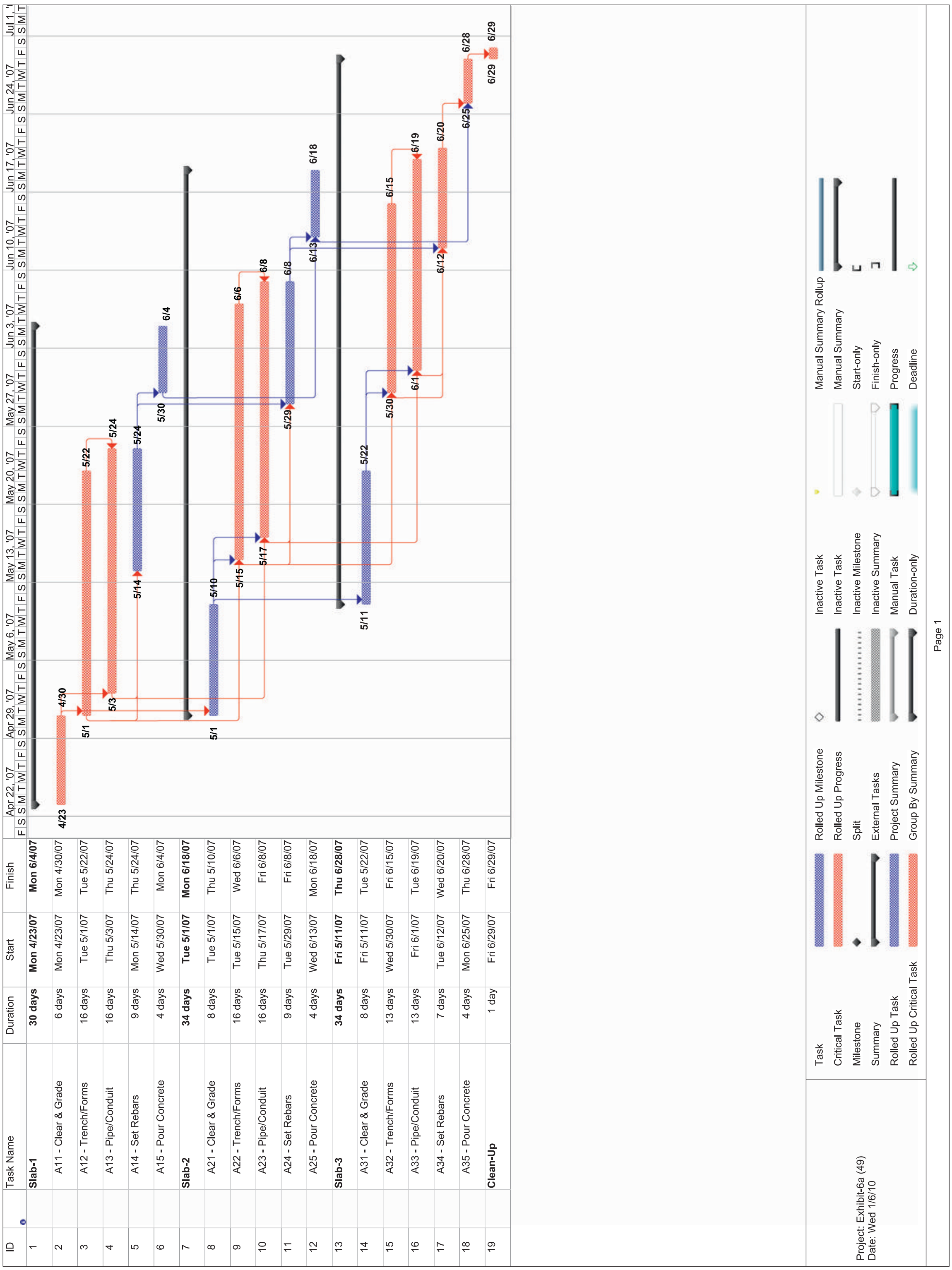

ID Task Name Duration Start Finish
1 Slab-1 30 days Mon 4/23/07 Mon 6/4/07
2 A11 - Clear & Grade 6 days Mon 4/23/07 Mon 4/30/07
3 A12 - Trench/Forms 16 days Tue 5/1/07 Tue 5/22/07
4 A13 - Pipe/Conduit 16 days Thu 5/3/07 Thu 5/24/07
5 A14 - Set Rebars 9 days Mon 5/14/07 Thu 5/24/07
6 A15 - Pour Concrete 4 days Wed 5/30/07 Mon 6/4/07
7 Slab-2 34 days Tue 5/1/07 Mon 6/18/07
8 A21 - Clear & Grade 8 days Tue 5/1/07 Thu 5/10/07
9 A22 - Trench/Forms 16 days Tue 5/15/07 Wed 6/6/07
10 A23 - Pipe/Conduit 16 days Thu 5/17/07 Fri 6/8/07
11 A24 - Set Rebars 9 days Tue 5/29/07 Fri 6/8/07
12 A25 - Pour Concrete 4 days Wed 6/13/07 Mon 6/18/07
13 Slab-3 34 days Fri 5/11/07 Thu 6/28/07
14 A31 - Clear & Grade 8 days Fri 5/11/07 Tue 5/22/07
15 A32 - Trench/Forms 13 days Wed 5/30/07 Fri 6/15/07
16 A33 - Pipe/Conduit 13 days Fri 6/1/07 Tue 6/19/07
17 A34 - Set Rebars 7 days Tue 6/12/07 Wed 6/20/07
18 A35 - Pour Concrete 4 days Mon 6/25/07 Thu 6/28/07
19 Clean-Up 1 day Fri 6/29/07 Fri 6/29/07
Apr 22, '07
Apr 29, '07
May 6, '07
May 13, '07
May 20, '07
May 27, '07
Jun 3, '07
Jun 10, '07
Jun 17, '07
Jun 24, '07
Jul 1, '
Project: Exhibit-6a (49)
Date: Wed 1/6/10
Task
Critical Task
Milestone
Summary
Rolled Up Task
Rolled Up Critical Task
Rolled Up Milestone
Rolled Up Progress
Split
External Tasks
Project Summary
Group By Summary
Inactive Task
Inactive Task
Inactive Milestone
Inactive Summary
Manual Task
Duration-only
Manual Summary Rollup
Manual Summary
Start-only
Finish-only
Progress
Deadline
Page 1

Exhibit 6a

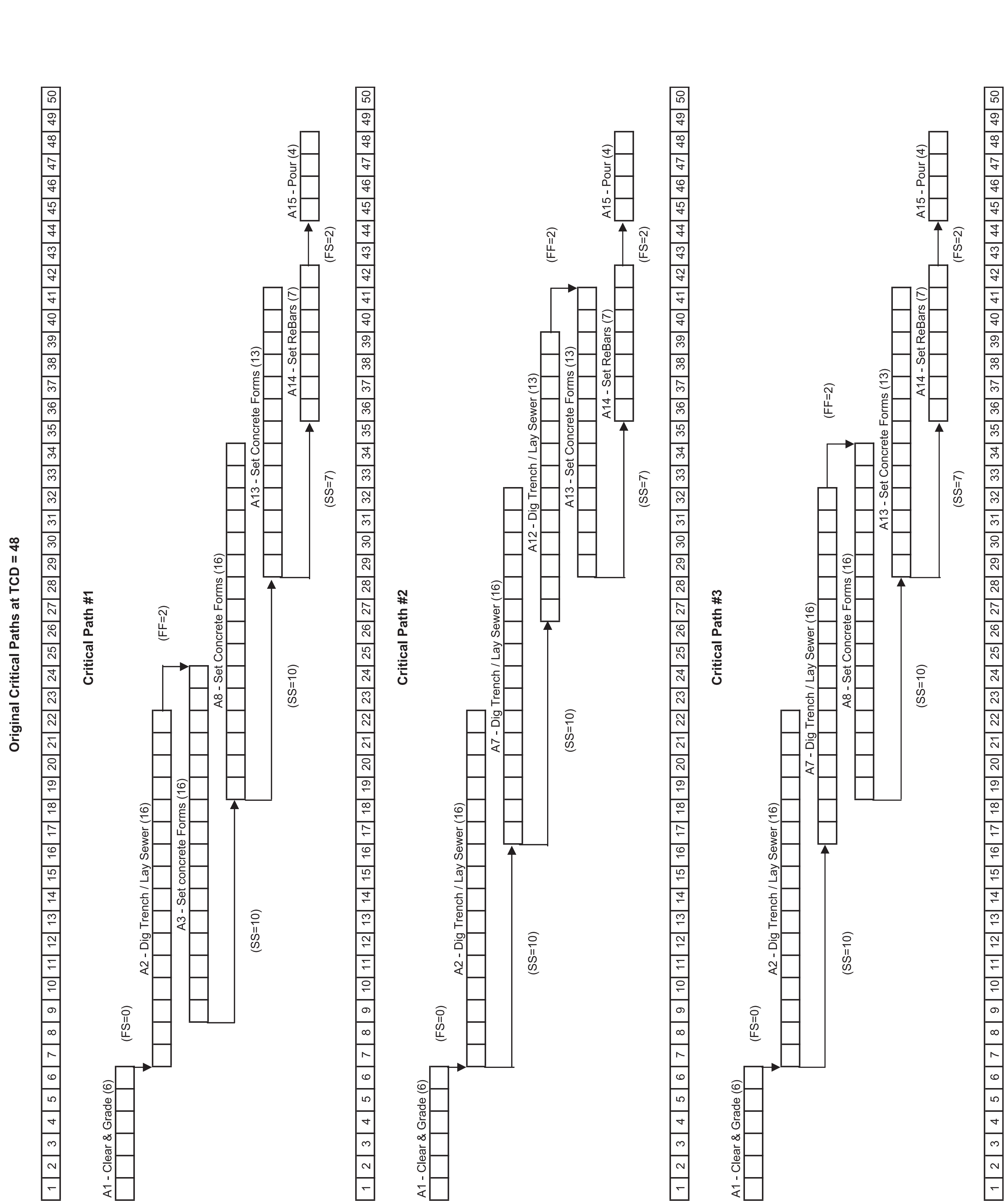

Original Critical Paths at TCD = 48
Critical Path #1
A1 - Clear & Grade (6)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(FF=2)
A3 - Set concrete Forms (16)
(SS=10)
A8 - Set Concrete Forms (16)
(SS=10)
A13 - Set Concrete Forms (13)
(SS=7)
A14 - Set ReBars (7)
(FS=2)
A15 - Pour (4)
Critical Path #2
A1 - Clear & Grade (6)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(SS=10)
A7 - Dig Trench / Lay Sewer (16)
(SS=10)
A12 - Dig Trench / Lay Sewer (13)
(FF=2)
A13 - Set Concrete Forms (13)
(SS=7)
A14 - Set ReBars (7)
(FS=2)
A15 - Pour (4)
Critical Path #3
A1 - Clear & Grade (6)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(SS=10)
A7 - Dig Trench / Lay Sewer (16)
(FF=2)
A8 - Set Concrete Forms (16)
(SS=10)
A13 - Set Concrete Forms (13)
(SS=7)
A14 - Set ReBars (7)
(FS=2)
A15 - Pour (4)

Exhibit 6b

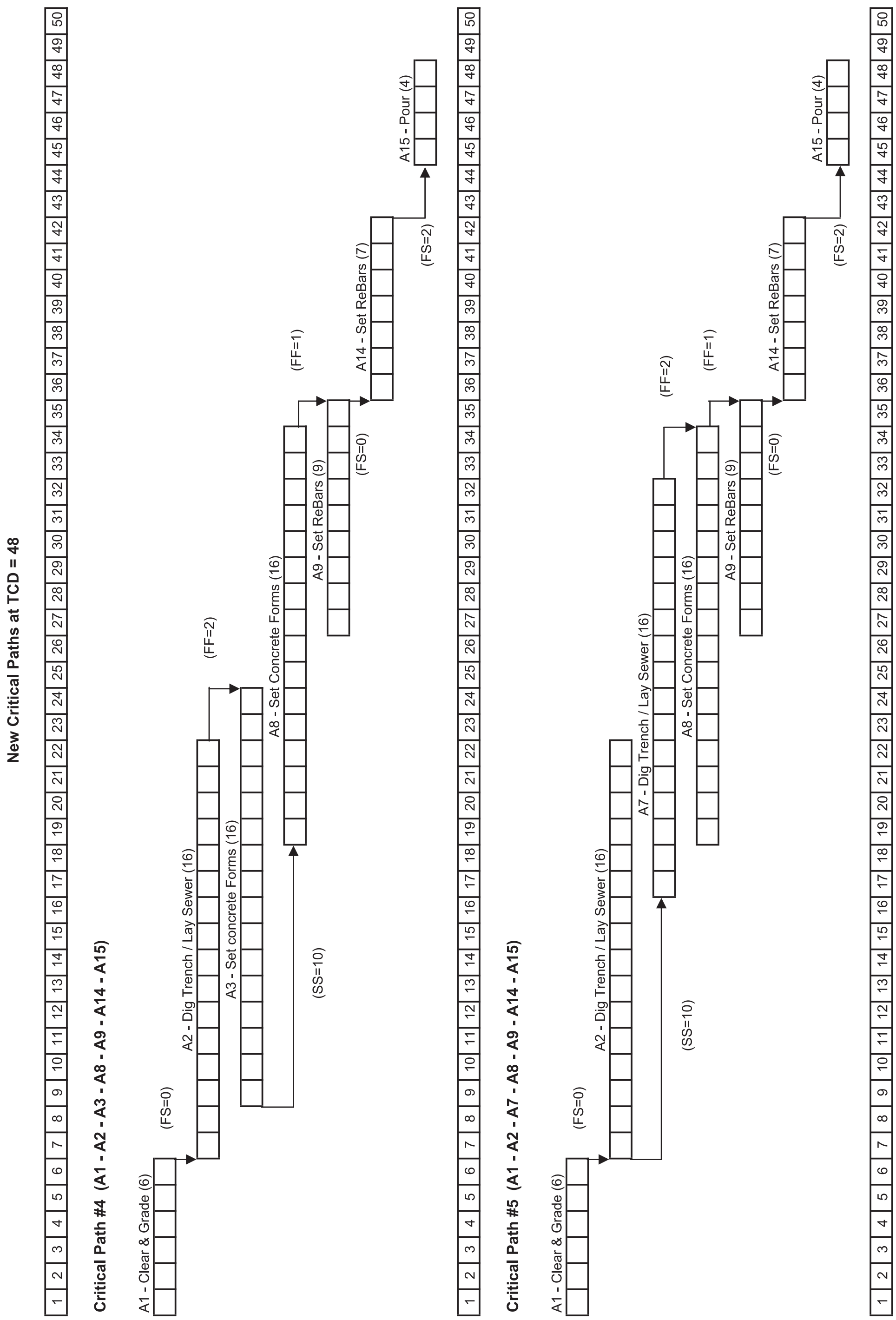

New Critical Paths at TCD = 48
Critical Path #4 (A1 - A2 - A3 - A8 - A9 - A14 - A15)
A1 - Clear & Grade (6)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(FF=2)
A3 - Set concrete Forms (16)
(SS=10)
A8 - Set Concrete Forms (16)
(FF=1)
A9 - Set ReBars (9)
(FS=0)
A14 - Set ReBars (7)
(FS=2)
A15 - Pour (4)
Critical Path #5 (A1 - A2 - A7 - A8 - A9 - A14 - A15)
A1 - Clear & Grade (6)
(FS=0)
A2 - Dig Trench / Lay Sewer (16)
(SS=10)
A7 - Dig Trench / Lay Sewer (16)
(FF=2)
A8 - Set Concrete Forms (16)
(FF=1)
A9 - Set ReBars (9)
(FS=0)
A14 - Set ReBars (7)
(FS=2)
A15 - Pour (4)

**Exhibit 6c**

project example was solved using the EXCEL Solver tool, and the solution to our scheduling dilemma is presented and summarized in Exhibit C2a. The results can be summarized as follows:

In order to accelerate the completion of the slab construction project from 56 to 48 days the least expensive plan is to:

(a) reduce the duration of activities (1) and (14) by 2 days,
(b) reduce the duration of activity (12) by 3 days,
(c) increase the duration of activities (3) and (8) by 4 days, and
(d) increase the duration of activity (13) by 1 day.

This project acceleration (time compression) plan is estimated to cost an additional $175.

The results of the time/cost tradeoff analysis were then used to edit the original data entered in Microsoft Project in order to reflect the conditions mandated by the client's insistence on compressing the total project duration from 56 to 48 days in order to complete the project prior to the end of the fiscal year. The updated Gantt Chart view for the new project schedule is presented in Exhibit 6a.

In comparing the original project schedule (Exhibit 3a) with the revised project schedule (Exhibit 6a), one can clearly see how the changes in activity durations affected the total project duration. For example, A1 was shortened by 2 days, which resulted in its planned completion being scheduled on 4/30 rather than 5/2. This allowed A2, whose duration remained at 16 days to be scheduled to start on 5/1 rather than 5/3, and have its scheduled completion on 5/22 rather than 5/24. As we compare the scheduled start and end date for activity A3 and the start date for A4 the anomaly becomes apparent. There is an FF2 precedence relationship between A2 and A3, which means that the revised completion date for A3 is 5/24 rather than 5/29 (only two working days as 5/26 thru 5/28 are non-working days—5/28 was Memorial day). However, the duration of A3 was increased by 4 days, which resulted in its scheduled start being moved forward from 5/9 to 5/3. Since the precedence relationship between A3 and A8 is SS10, the scheduled start date for A8 is "pulled" forward 4 days—and A8 is lengthened by 4 days because of its FF2 relationship with A7. The interaction effects of the various types of precedence in the PDM become readily apparent during continued reconciliation of the original and revised project schedules.

Further, as is the case in traditional time/cost tradeoff analysis, shortening the total project duration typically causes activities that were originally non-critical to become critical and creates new critical paths. In our example, Activity (9) has become critical as the total project duration was decreased from 56 to 48 days. Based on the precedence relationships between Activity (9) and other project activities, two new critical paths were created. The three original critical paths are recast in Exhibit 6b, and the two new critical paths are illustrated in Exhibit 6c.

The planned project schedule has been revised to meet the requirements of the client at minimum cost. This was accomplished by combining the capabilities of Microsoft Project (a contemporary tool) with our original LP formulation of the time/cost tradeoff problem for precedence diagrams (a traditional tool).

**SLAB CONSTRUCTION PROJECT / Accelerated Solution**

TCD = 48    Additional Cost = $175

| Activity | Description | T(j) | E(j) | L(j) | Y(j) | C(j) |
|---|---|---|---|---|---|---|
| A1 | Clear 1 | 6 | 1 | 6 | -2 | 50 |
| A2 | Sewer 1 | 16 | 7 | 22 | 0 | 0 |
| A3 | Forms 1 | 16 | 9 | 24 | +4 | 20 |
| A4 | Rebars 1 | 9 | 17 | 26 | 0 | 0 |
| A5 | Pour 1 | 4 | 28 | 44 | 0 | 0 |
| A6 | Clear 2 | 8 | 7 | 16 | 0 | 0 |
| A7 | Sewer 2 | 16 | 17 | 32 | 0 | 0 |
| A8 | Forms 2 | 16 | 19 | 34 | +4 | 20 |
| A9 | Rebars 2 | 9 | 27 | 35 | 0 | 0 |
| A10 | Pour 2 | 4 | 38 | 46 | 0 | 0 |
| A11 | Clear 3 | 8 | 15 | 26 | 0 | 0 |
| A12 | Sewer 3 | 13 | 27 | 39 | -3 | 60 |
| A13 | Forms 3 | 13 | 29 | 41 | +1 | 5 |
| A14 | Rebars 3 | 7 | 36 | 42 | -2 | 20 |
| A15 | Pour 3 | 4 | 45 | 48 | 0 | 0 |
| | | | | | | 175 |

**Exhibit C2**

## 4. Summary

This paper illustrates the successful integration of contemporary (Microsoft Project) and traditional (a linear programming time/cost tradeoff model) project management tools to solve a hypothetical, yet realistic, scheduling problem for projects modeled in the precedence diagramming format. The initial schedule was developed using Microsoft Project and validated using standard PDM critical path analysis. When there was a need to compress (shorten) the initial schedule because of constraints imposed by the client, it was necessary to develop a linear programming time/cost tradeoff model for the precedence diagramming environment to ensure resolution at minimum cost.

The linear programming formulation (model) for the time/cost tradeoff problem for projects modeled using the precedence diagramming method, which was developed and introduced in this paper, is a new contribution to the project management literature. Further, the resolution of the example problem clearly demonstrates the anomalous effects that can occur when specific combinations of (FF) and (SS) precedence relationships are present. More specifically, the example demonstrates that when projects are modeled and managed using the precedence structures available in the precedence diagramming method, it may be necessary to lengthen one or more activities in order to shorten the overall duration of the project.

While the example problem is relatively straightforward, this paper demonstrates the viability of combining both contemporary and traditional tools to resolve project management issues, and hopefully will provide insight for extensions of this approach as might be required by more complex projects—particularly when they are modeled in the precedence diagramming format.

## Appendix A. Notation

In this section the notation used in the specification of the algorithm for critical path analysis of precedence diagrams (PDM project networks) and the formulation of our time/cost tradeoff linear programming model is presented.

Let:

$E(j)$ early start time of activity $j$
$L(j)$ late finish time of activity $j$
$T(j)$ actual duration of activity $j$, $=N(j)\pm Y(j)$, where
$N(j)$=normal duration of activity $j$
$Y(j)$=change (increase/decrease) in duration of activity $j$
$M(j)$ maximum decrease in duration of activity $j$, thus $Y^-(j)\le M(j)$
$C^-(j)$ marginal cost of decreasing the duration of activity $j$
$C^+(j)$ marginal cost of increasing the duration of activity $j$
$i$ an index used to represent activities that are immediate predecessors of activity $j$

$k$ an index used to represent activities that are immediate successors of activity $j$

During the development of the equations for the time/cost tradeoff model, it will also be necessary to develop expressions for the early finish time and late start time for project activities. Straightforward logic reveals that:

the early finish time for activity $(j) = E(j) + T(j) = E(j) + N(j) \pm Y(j)$,

and

the late start time for activity $(j) = L(j) - T(j) = L(j) - N(j) \pm Y(j)$.

As was indicated in Exhibit 1, we will also employ the following notation for the lead/lag factors associated with the various types of precedence relationships:

$A(i,j)$ = lead/lag factor for finish-to-start (FS) relationships

$B(i,j)$ = lead/lag factor for start-to-start (SS) relationships

$C(i,j)$ = lead/lag factor for finish-to-finish (FF) relationships

$D(i,j)$ = lead/lag factor for start-to-finish (SF) relationships

## Appendix B. Conventional critical path analysis of projects modeled in the precedence diagramming format

*The Forward Pass*:

If the precedence relationship between activities $(i,j)$ is of the "finish-to-start" (FS) type, the early start time of activity $(j)$ must be greater than or equal to the early finish time of activity $(i)$ plus any specified offset, i.e., $A(i,j)$. Thus, the early start time for activity $(j)$ must satisfy the following expression:

$[E(j)|FS] \geq E(i) + N(i) + A(i,j)$

If the precedence relationship between activities $(i,j)$ is of the "start-to-start" (SS) type, the early start time of activity $(j)$ must be greater than or equal to the early start time of activity $(i)$ plus any specified offset, i.e., $B(i,j)$. Thus, the early start time for activity $(j)$ must satisfy the following expression:

$[E(j)|SS] \geq E(i) + B(i,j)$

If the precedence relationship between activities $(i,j)$ is of the "finish-to-finish" (FF) type, the early finish time of activity $(j)$ must be greater than or equal to the early finish time of activity $(i)$ plus any specified offset, i.e., $C(i,j)$. Thus, the early start time for activity $(j)$ must satisfy the following expression:

$[E(j)|FF] \geq E(i) + N(i) - N(j) + C(i,j)$

Although the "start-to-finish" (SF) type of precedence is not often encountered in actual practice [7], if the precedence relationship between activities $(i,j)$ is of the (SF) type, the early finish time of activity $(j)$ must be greater than or equal to the early start time of activity $(i)$ plus any specified offset, i.e., $D(i,j)$. Thus, the early start time of activity $(j)$ must satisfy the following relationship:

$[E(j)|SF] \geq E(i) - N(j) + D(i,j)$

and

$E(j) = \text{MAX}[[E(j)|FS], [E(j)|SS], [E(j)|FF], [E(j)|SF]]$ for all $(i)$ preceding $(j)$

*Backward Pass*:

If the precedence relationship between activities $(j,k)$ is of the "finish-to-start" (FS) type, the late finish time of activity $(j)$ must be less than or equal to the late start time of activity $(k)$ minus any specified offset, i.e., $A(i,j)$. Thus, the late finish time for activity $(j)$ must satisfy the following expression:

$[L(j)|FS] \leq L(k) - N(k) - A(j,k)$

If the precedence relationship between activities $(j,k)$ is of the "start-to-start" (SS) type, the late start time of activity $(j)$ must be less than or equal to the late start time of activity $(k)$ minus any specified offset, i.e., $B(j,k)$. Thus, the late finish time for activity $(j)$ must satisfy the following expression:

$[L(j)|SS] \leq L(k) - N(k) + N(j) - B(j,k)$

If the precedence relationship between activities $(j,k)$ is of the "finish-to-finish" (FF) type, the late finish time of activity $(j)$ must be less than or equal to the late finish time of activity $(k)$ minus any specified offset, i.e., $C(j,k)$. Thus, the late finish time for activity $(j)$ must satisfy the following expression:

$[L(j)|FF] \leq L(k) - C(j,k)$

If the precedence relationship between activities $(j,k)$ is of the "start-to-finish" (SF) type, the late start time of activity $(j)$ must be less than or equal to the late finish time of activity $(k)$ minus any specified offset, i.e., $D(j,k)$. Thus, the late finish time for activity $(j)$ must satisfy the following expression:

$[L(j)|SF] \leq L(k) + N(j) - D(j,k)$

and

$L(j) = \text{MIN}[[L(j)|FS], [L(j)|SS], [L(j)|FF], [L(j)|SF]]$ for all $(k)$ succeeding $(j)$

## Appendix C1. Linear programming formulation of basic critical path analysis for projects modeled in the precedence diagramming format

*Model Specifications*:

The linear programming formulation for the standard critical path analysis of a project modeled in the precedence diagramming format is relatively efficient and easily specified. There are:

| | | |
|---|---|---|
| (a) | two variables for each activity | $E(j)$=early start time for activity $(j)$<br>$L(j)$=late finish time for activity $(j)$ |
| (b) | one constraint for each activity | $L(j) - E(j) \geq N(j)$ |
| (c) | two constraints for each precedence relationship: | Forward pass/early<br>Backward pass/late |
| Note: | In the precedence diagramming format it is possible to specify more than one type of precedence between a single pair of activities. | |

The standard critical path analysis model for the PDM format is specified as follows:

| | | |
|---|---|---|
| Objective function | MIN $Z = L(TN) + \sum E(j) - \sum L(j)$, where TN is the project termination node | |
| Subject to | $L(j) - E(j) \geq N(j)$ | for all $(j)$ |
| For FS relationships | $E(j) - E(i) \geq N(i) + A(i,j)$<br>$L(k) - L(j) \geq N(k) + A(j,k)$ | |
| For SS relationships | $E(j) - E(i) \geq B(i,j)$<br>$L(k) - L(j) \geq N(k) - N(j) + B(j,k)$ | |

For FF relationships $E(j)-E(i) \geq N(i)-N(j)+C(i,j)$
$L(k)-L(j) \geq C(j,k)$

$E(j)$, $L(j) \geq 0$, for all $j$

Note: In the critical path analysis of precedence diagrams the objective function is simply: MIN $L(\text{TN})+\sum E(j)-\sum L(j)$, where TN is the index of the last activity in the network. The $\sum E(j)$ term is included to ensure that the early start times for non-critical activities are set to their minimum value and the $-\sum L(j)$ term is included to ensure that the late finish times of the non-critical activities are set to their maximum value.

## Appendix C2. Linear programming formulation of time/cost tradeoff analysis for projects modeled in the precedence diagramming format

The linear programming formulation of a time/cost tradeoff model for the precedence diagramming format is a bit more complex than the basic critical path analysis model. There are:

(a) four variables for each activity
- $E(j)$=early start time for activity ($j$)
- $L(j)$=late finish time for activity ($j$)
- $Y^-(j)$=decrease in duration for activity ($j$)
- $Y^+(j)$=increase in duration for activity ($j$)

(b) two constraints for each activity
- $Y^-(j) \leq M(j)$
- $L(j)-E(j) \geq N(j)+Y^+(j)-Y^-(j)$

(c) two constraints for each precedence relationship
- Forward pass/early
- Backward pass/late

Note: Because of the possibility of "reverse" criticality, the change in duration variable [$Y$] must be split, i.e., $Y=Y^+-Y^-$ to allow for both decreases and increases in an activity duration, [18] and [19].

Suppose that we are required to complete the Slab Construct Project in 48 days instead of the NPD of 56 days. The formulation of the time/cost tradeoff model for the Slab Construct Project contains 60 variables (4 for each of the 15 activities), and 98 constraints (2 for each of 15 activities plus 2 for each of 34 precedence relationships). The linear programming model for the time/cost tradeoff problem formulation is specified as follows:

MIN $Z=\sum [C^-(j)^*Y^-(j)]+C^{+*}\sum Y^+(j)$
Subject to
$L(\text{TN})=\text{TCD}$
$Y^-(j) \leq M(j)$, for all ($j$)
$L(j)-E(j) \geq N(j)-Y^-(j)+Y^+(j)$, for all ($j$)

For FS relationships
$E(j)-E(i)+Y^-(i)-Y^+(i) \geq N(i)+A(i,j)$
$L(k)-L(j)+Y^-(k)-Y^+(k) \geq N(k)+A(j,k)$

For SS relationships
$E(j)-E(i) \geq B(i,j)$
$L(k)-L(j)+Y^-(k)-Y^+(k)-Y^-(j)+Y^+(j) \geq N(k)-N(j)+B(j,k)$

For FF relationships
$E(j)-E(i)-Y^-(j)+Y^+(j)+Y^-(i)-Y^+(i) \geq N(i)-N(j)+C(i,j)$
$L(k)-L(j) \geq C(j,k)$

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