



# Fresh-product supply chain management with logistics outsourcing

Xiaoqiang Cai<sup>a</sup>, Jian Chen<sup>b,\*</sup>, Yongbo Xiao<sup>b</sup>, Xiaolin Xu<sup>c</sup>, Gang Yu<sup>d</sup>

<sup>a</sup> Department of Systems Engineering and Engineering Management, The Chinese University of Hong Kong, NT, Hong Kong

<sup>b</sup> Research Center for Contemporary Management, Key Research Institute of Humanities and Social Sciences at Universities, School of Economics and Management, Tsinghua University, Beijing 100084, China

<sup>c</sup> School of Business, Nanjing University, Nanjing 210093, China

<sup>d</sup> Department of Management Science and Information Systems, The University of Texas at Austin, USA

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## ABSTRACT

We consider a supply chain in which a producer supplies a fresh product, through a third-party logistics (3PL) provider, to a distant market where a distributor purchases and sells it to end customers. The product is perishable, both the quantity and quality of which may deteriorate during the process of transportation. The market demand is random, sensitive to the selling price as well as the freshness of the product. We derive the optimal decisions for the three supply chain members, including the 3PL provider's transportation fee, the producer's shipping quantity and wholesale price, and the distributor's purchasing quantity and retail price. We find that the presence of the 3PL provider in the supply chain has a significant impact on its performance. We propose an incentive scheme to coordinate the supply chain. The scheme consists of two contracts, including a wholesale-market clearance (WMC) contract between the producer and the distributor, and a wholesale-price-discount sharing (WDS) contract between the producer and the 3PL provider. We show that the proposed contracts can eliminate the two sources of "double marginalization" that exist in the three-tier supply chain, and induce the three parties to act in a coordinated way.

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## 1. Introduction

We consider a supply chain in which a fresh-product producer supplies the product to a distant market, via a specialized third-party logistics (3PL) provider, where a distributor purchases and sells it to end customers. Because of the vast distance between the production base and the target market, the transport time is long and usually quite unstable. As a result, the fresh product is prone to decay/deterioration during the process of transportation. Moreover, end customers are sensitive to both the retail price and level of freshness of the product, and thus the market demand is random, highly depending on these two factors. With uncertainties in transport time, level of freshness, and market demand, the decisions of the three parties involved in such a supply chain are complicated, which may cause great losses if not made appropriately. The main purpose of this paper is to develop a model to address these issues, to characterize the optimal decisions that each party should adopt, and to examine the appropriate incentive schemes to motivate the chain members to coordinate so that everyone benefits from the improved performance of the system.

Different structures exist in fresh product supply chains, depending on how parties such as producers, collectors, brokers, wholesales, and retailers, etc., are involved. Cadilhon et al. [9, p. 137] summarize five typical structures. The model we consider in this paper corresponds to one of the two modern distribution systems (Structure 5 of Cadilhon et al. [9]), which represents a direct distribution channel from the producer to the retailer. One example that supports our model is the Floratrading business developed in Ecuador's cut flower industry [7], which has been set up to capture the market opportunities in rural regions of America. The development gives rise to a fully integrated supply chain for roses, involving a grower-owned brokerage firm, Floratrading, located in the production base (which we call the "Producer" in our model), UPS for the logistics and transportation (the "3PL Provider" in our model), and a rural florist (which we call the "Distributor") to sell the product to end customers in the American market. Fig. 1 of [7] shows the new distribution channel consisting of Floratrading, UPS, and rural florist, in comparison with other more traditional channels. Another example that has motivated our work is Kunming Hongri Flower Plant Co., Ltd. ([kunming-hongri.en.ywsp.com](http://kunming-hongri.en.ywsp.com)), a specialized export company of fresh cut flowers that locates in Kunming, one of the biggest flower plant bases in the world. The firm exports carnations, roses, lilies, etc., to other countries (including Japan and South Korea), through specialized 3PL providers. The market demand of

\* Corresponding author. Tel.: +86 10 62789896; fax: +86 10 62771647.  
E-mail address: [chenj@sem.tsinghua.edu.cn](mailto:chenj@sem.tsinghua.edu.cn) (J. Chen).

the fresh-cut flower depends heavily on its freshness upon arriving at the destination markets. Therefore, how to maintain the quality of the flower is a key concern in their operations. Although motivated by the practices of fresh-flower supply chains, the model we study is also applicable to other problems that involve production, transportation, and distribution of fresh produces, including fruit, vegetables, live seafood, etc.

Long distance transportation is inevitable in most fresh-product supply chains due to the geographic separation of the production base and the target market. Because of the requirements on long-haul delivery and freshness keeping, transportation logistics is often outsourced to specialized logistics providers with the needed capacity and facility. In this paper, we are interested to understand how the involvement of the 3PL provider would impact the supply chain, in particular the corresponding decisions to be taken by parties concerned. On the one hand, the capability of long-distance shipping with the necessary cooling facility enables the 3PL provider to gain an advantage over other means of transportation. On the other hand, however, the 3PL provider still needs to consider the reactions of the producer while negotiating on the transportation fee and other clauses with the producer. As it is well known, carriers charge three types of rates: published, counter, and negotiated [16]. However, as stressed by Sanfilippo [37], no business firms should accept published shipping rates, and it is common that 3PL providers such as UPS, FedEx, and DHL offer shipment-specific pricing contracts to industrial shippers.

Variation of transport time can be very large for long-distance transportation. For instance, according to Vega [47], “a shipment of fresh flowers, from the time of harvest on a farm located near Quito until the moment it arrives to a U.S. retailer, can take from 44  $\frac{1}{2}$  hours to almost 13 days.” A large time delay can cause significant loss in value of fresh products (it is reported that most bouquets last up to 7–10 days if kept cool; see, e.g., [www.gardenguides.com](http://www.gardenguides.com)). Considering the uncertainties in transport time, how should the 3PL provider determine the transportation fee? How will the pricing of the 3PL provider affect the decisions of the producer and other players in the supply chain? Both [36,47] have pointed out, by statistic analysis, that transport costs are a significant component of the final prices for fresh products. Thus, how should the producer take into account the transport cost and time, to make the most appropriately decisions? How would these affect the decisions of the downstream distributor? Could the three parties be motivated to take the coordinated decisions, so that the performance of the entire supply chain is optimized and consequently everyone benefits? Answers to these and other related questions are important for understanding the supply chain and the corresponding strategies and decisions its members should take, which we will investigate in this paper.

The remainder of the paper is organized as follows. In Section 2 we provide a brief review of the related literature. The problem formulation, assumptions, and notation are presented in Section 3. In Section 4, we characterize the optimal decisions of the three parties in the decentralized system. Optimal decisions in the fully centralized system and partially centralized systems are investigated in Section 5. Section 6 develops an incentive scheme to coordinate the decentralized system. Section 7 summarizes our work, where topics for future study are also discussed.

## 2. Related literature

One stream of the literature related to our research is on logistics outsourcing. This is a business strategy that has been widely adopted in practice and studied in the literature over the past two decades. For example, [40] develops a theoretical

framework, including both transaction cost theory and network theory, to explain the role and motivation of third-party outsourcing arrangements. Tyan et al. [45] examine a special class of freight consolidation policies of a 3PL provider that seeks to maximize the utilization of expensive transportation such as aircraft. Vaidyanathan [46] explores the major considerations in searching for a 3PL provider and develops an evaluation framework. Fong [19] presents three new models for logistics network design with special focus on the perspective of 3PL companies. More discussion on outsourcing of logistical activities can be found in review papers by Lieb [27], McKinnon [28], Razzaque and Sheng [35], and Sheffi [39]. As can be seen, most of the literature considers issues on certain aspects of logistics outsourcing, whereas interactions between decisions of 3PL providers and their clients are not addressed [4]. Song et al. [41] is one of the few papers that study the decision problems faced by 3PL providers in a supply chain system. They focus on the scheduling problem of a 3PL provider, who needs to coordinate shipments between suppliers and customers through a consolidation center in a distribution network. Our model studies the 3PL provider's pricing decision and its impact on the decisions of other firms in the supply chain with a time-sensitive fresh product.

Research on supply chain management of perishable products is another stream of literature related to our research. Whittin [53] studied a perishable inventory problem in which fashion goods deteriorated at the end of certain storage periods. Since then, considerable attention has been paid to this line of research. Nahmias [29] provides a comprehensive survey of the literature published before the 1980s, in which perishable products with fixed lifetime and random lifetime were categorized. More recent studies on deteriorating inventory models can be found in Raafat [33], Goyal and Giri [20], Ferguson and Koenigsberg [18], Ketzenberg and Ferguson [22], and Blackburn and Scudder [6]. Kopanos et al. [23] consider the problem of simultaneous production and logistics planning in food industries. An integrated mixed integer programming model is developed, which incorporates various practical factors and constraints. Post-production perishability of food products is, however, not specifically considered in their model. Wang and Li [51] investigate different pricing policies based on dynamically identified food quality, in order to reduce food spoilage waste and maximize food retailer's profit.

Generally, two types of perishable loss, quantity loss and quality loss, may take place for a perishable product. The majority of the literature has dealt mainly with only one type of loss. One exception is Rajan et al. [34], who consider both value drop and quantity decrease. However, they focus on a model with deterministic demand, in which the decision maker aims to optimize the selling price and the order cycle length of inventory replenishment to maximize the average profit per unit time. Our model considers a fresh product subject to both types of loss during transportation: the quantity decrease affects the effective supply when the product reaches the market, and the quality deterioration affects the market demand. Both impacts are captured in our model using general functions of the actual transportation time. The fact that the market demand depends on the level of freshness of the product and that the freshness depends on the transport time makes the decision making of the producer a two-stage problem, because the wholesale pricing decision relies on the actual level of freshness after transportation. This is in sharp contrast with the literature on decentralized supply chain management, in which the upstream supplier only makes a one-stage decision (see, e.g., [2,5]).

Coordination of the three parties involved in a fresh-product supply chain – the producer, the 3PL provider, and the distributor – is a main subject addressed in this paper. Coordination of two parties, usually a supplier and a distributor (or a retailer), has been a subject of extensive study in the supply chain management field

over the last few decades (see, e.g., Chen, [12]). Such coordination is often achieved through contracts between the upstream supplier and the downstream distributor, to increase the total supply chain profit and make it closer to the profit that can be generated from a centralized control (channel coordination), or to share risk among supply chain partners [43]. Various models of supply chain contracts have been developed. Price discounts are often suggested as incentives to facilitate coordination (see Parlar and Wang, [30], Weng, [52], Wang, [48]). Other incentive schemes include quantity commitment [2], quantity flexibility contracts [43], backup agreements (Eppen and Iyer, [17]), buy-back or return policies (Pasternack, [31]), revenue sharing (Cachon and Larivière, [8]), sales rebates or markdown allowances (Krishnan et al., [24]). Some recent research studies supply chain contracts considering the integration of production and other functional departments. For example, Caldentey and Haugh [11] study a supply contract with financial hedging. Chick et al. [13] design a variant of the cost sharing contract to coordinate the influenza vaccine supply chain. Choi et al. [14] consider a two-echelon supply chain with one manufacturer producing a fashionable product and one retailer, under the assumption that each chain member is concerned about a risk as expressed as the variance of his profit. Conditions on channel coordination are evaluated.

Research on supply chain coordination among multiple members is relatively scarce in the literature, due to the difficulties associated with the possible contracts. To the best of our knowledge, only a few papers have considered coordination mechanisms among more than two chain members. Leng and Parlar [25] consider a three-level supply chain with demand information sharing among a manufacturer, a distributor, and a retailer. They apply a cooperative game approach to develop cost savings allocation schemes too facilitate cooperation among the three players. Shang et al. [38] study a periodic-review,  $N$ -stage serial supply chain in which materials are ordered and shipped according to  $(R,nQ)$  policies. They propose coordination schemes that regulate the stages to achieve the optimal cost of the supply chain under three information scenarios (echelon, local, and quasi-local). Ding and Chen [15] study the coordination issue of a three-level supply chain selling short life-cycle products in a single period model. They construct a flexible return policy by setting the rules of pricing and postponing the determination of the final contract prices to induce the three firms to act in a coordinated manner. By incorporating the 3PL provider as a major player, the structure of our supply chain is quite different from those reviewed above. Our proposed coordination scheme consists of two contracts: a contract between the 3PL provider and the producer, and a contract between the producer and the distributor. This is very different from the schemes developed in the existing literature.

Cai et al. [10] have recently studied a fresh-product supply chain in which the *downstream distributor* is responsible for the long-distance transportation. This is different from the business model we consider in the present paper, where the transportation is to be undertaken by the *upstream producer* (through outsourcing to a 3PL provider). The difference between the two business models makes the decision problems faced by the producer and the distributor rather different. For examples, (i) in Cai et al. [10], the quantity of product to be shipped is determined by the distributor, whereas in the model studied in this paper, it is determined by the producer; and (ii) the wholesale price is determined before the transportation in Cai et al. [10], whereas in this paper it is determined after the product has arrived at the distant market according to the actual freshness level then. As one may see, there are two sources of “double marginalization” in the three-tire supply chain considered in this paper. This makes the development of a coordination mechanism much more complicated.

### 3. The model

We are concerned about the following problem. A producer ships a batch of fresh product, through a 3PL provider, to a distant market, where a downstream distributor (or a retailer) purchases and resells the product to end customers. By outsourcing the shipment of the product to a 3PL provider, the producer is responsible for the transportation cost and bears the risk of product decay/deterioration during transportation. After the product arrives at the market, the producer may decide on the actual wholesale price to be offered to the distributor (equivalently, decide on if any price discount should be provided), according to the level of freshness and the surviving quantity of the product.

Assume that the producer's unit production cost is  $c_1$ . The product is fully fresh when loaded onto the vehicle (e.g., a cargo ship). It remains fresh during a period that we call its fresh-duration,  $\tau$ , which depends on the nature of the product and the way in which it is treated and stored [21]. After that, the product starts to perish at a significant rate. The perishability may lead to “deterioration” and “obsolescence”, both of which can occur during transportation. Deterioration lessens the quality (freshness) of the product, and obsolescence reduces the surviving quantity. Specifically, we model the two types of perishability by the following two time-dependent indices, where  $t=0$  is the time at which the product is loaded on the vehicle:

- A function  $\theta(t)$  of time  $t$ , defined over  $[0,1]$ , as the freshness index of the product:  $\theta(t) = 1$  if  $t \leq \tau$  and  $0 \leq \theta(t) < 1$  otherwise.
- A function  $m(t)$  of time  $t$ , defined over  $[0,1]$ , as the index on the surviving quantity of the product at time  $t$ . Suppose that  $q$  units of the product are loaded onto the vehicle, the surviving quantity becomes  $qm(t)$  after a period of time  $t$ , where  $0 < m(t) \leq 1$ .

Note that exponential functions have been used in the literature to model quantity decreases and quality declines of perishable products: see Raafat [33], Rajan et al. [34], and Blackburn and Scudder [6]. Our functions  $\theta(t)$  and  $m(t)$  can be any functions, depending on the nature of the product.

The market demand for the product depends on its freshness level  $\theta$  and the retail price  $p$  of the distributor, with the following multiplicative functional-form:

$$D(p, \theta) = y_0(\theta)p^{-k_0(\theta)} \cdot \varepsilon,$$

where  $y_0(\cdot)$  is the scaling factor that measures the potential market size,  $k_0(\cdot)$  is the price elasticity, and  $\varepsilon$  is a random variable that reflects the fluctuations of the market demand. Note that both  $y_0(\cdot)$  and  $k_0(\cdot)$  depend on the freshness level ( $\theta$ ) of the product. We let  $f(x)$  and  $F(x)$  to denote the PDF and CDF of  $\varepsilon$ , respectively. Without loss of generality, we assume  $E[\varepsilon] = 1$ ; this can be achieved by adjusting the scaling function  $y_0(\theta)$  accordingly.

**Assumption 1.** Suppose the demand function satisfies the following conditions:

- (i)  $y_0(\theta)$  is increasing in  $\theta$ ;
- (ii)  $k_0(\theta)$  is decreasing in  $\theta$ , with  $k_0(\theta) > 1$  for any given  $\theta$ ;
- (iii)  $\varepsilon$  has an increasing generalized failure rate (IGFR), and  $\lim_{x \rightarrow \infty} x[1 - F(x)] = 0$ .

Generally, the conditions in Assumption 1 reflect the fact that the size of market demand is positively correlated to the product's freshness level. As  $y_0(\theta)$  represents the potential market size, it is reasonable to assume that it is increasing in the freshness level  $\theta$ . Empirical studies have revealed that consumers are more willing

to buy a fresh product that has a longer expiration date [44]. We assume that the price elasticity  $k_0(\theta)$  is decreasing in freshness  $\theta$ , i.e., the larger the  $\theta$  value, the less sensitive the demand to a change in price. A product is defined as price-elastic if its price-elasticity is greater than 1, and as inelastic otherwise. Empirical studies have shown that many fresh products are price elastic; for example, fresh green peas and fresh tomatoes have a price-elasticity of 2.8 and 4.6 respectively [1]. We focus on price-elastic products and thus assume that  $k_0(\theta) > 1$  for any given  $\theta$ . As indicated by Rajan et al. [34], when the product becomes less fresh, a price discount should be offered to maintain the same level of demand. Note that if  $y_0(\theta)$  and  $k_0(\theta)$  are both constants (i.e., the product is not perishable), then the demand function reduces to that considered by Petruzzi and Dada [32], Wang [49], and Wang et al. [50].

Note that both the assumptions IGFR and  $\lim_{x \rightarrow \infty} x\bar{F}(x) = 0$  are mild restrictions on the distribution of  $\varepsilon$ . IGFR is a weaker condition than increasing failure rate, a property that is known to be satisfied by distributions such as normal, uniform, and the Gamma and Weibull families, subject to parameter restrictions [3]. The condition  $\lim_{x \rightarrow \infty} x\bar{F}(x) = 0$  is also satisfied by the aforementioned distribution functions.

Denote  $T$  as the transportation time, which is assumed to be a continuous random variable distributed over  $[a, b]$ , with CDF and PDF

functions being  $G(t)$  and  $g(t)$ , respectively. When  $b = a$ , our model reduces to the special case with a deterministic (fixed) transportation time. Assume that the 3PL provider's unit transportation cost is  $c_2$ . We consider the situation in which the transaction between the 3PL provider and the producer is on a shipment-by-shipment basis. That is, the transportation fee is a decision variable to be determined for each shipment, instead of being exogenously determined by the market. Suppose the 3PL provider charges a base transportation price  $s$  and offers a compensation to the producer if any transportation delay occurs. We assume the compensation rate is increasing in the realized transportation time  $t$ . As a result, the actual transportation fee charged by the 3PL provider is  $s\gamma(t)$ , where  $\gamma(t)$  is a decreasing function of the transportation time  $t$ . For example, if a logistics service provider offers to pay 0.3% of the freight as default penalty for every 1-day delivery delay, then  $\gamma(t) = 0.997^{(t-t_0)^+}$ , where  $t_0$  is the committed transportation time. Another example is the pricing tables of well-known logistics providers such as FedEx or UPS [42]. They apply different rates for different committed shipping times, which can be translated into the pricing model as we consider here. In our model,  $\gamma(t)$  can take any form: it can be either continuous or piece-wise continuous. For simplicity, we denote the expected value of  $\gamma(t)$  with respect to the transportation time as  $\gamma_0 > 0$ ; i.e.,  $\gamma_0 = \mathbb{E}\{\gamma(T)\}$ .

We assume the salvage value of any product left unsold is zero (because it is highly perishable), and we do not consider any

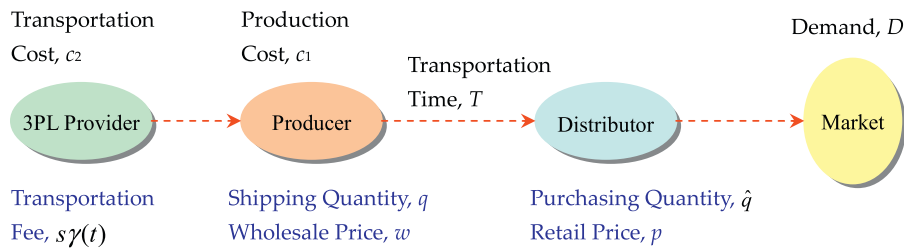


Fig. 1. The supply chain under consideration.

Table 1  
List of notation.

| Symbol         | Description  |
|----------------|--|
| $\tau$         | Fresh duration of the product  |
| $T$            | The transportation time distributed over $[a, b]$ , with PDF and CDF being $g(x)$ and $G(x)$   |
| $t$            | A realization of the random transportation time $T$ (i.e., the actual transportation time)   |
| $\theta(t)$    | Freshness index of the product w.r.t. the realized transportation time $t$   |
| $m(t)$         | Surviving quantity of the product w.r.t. the realized transportation time $t$  |
| $y_0(\theta)$  | The potential market size w.r.t. the freshness level $\theta$  |
| $k_0(\theta)$  | The price elasticity of the market demand w.r.t. the freshness level $\theta$  |
| $\varepsilon$  | Random fluctuation of the demand, with PDF, CDF, and complementary CDF being $f(\cdot)$ , $F(\cdot)$ and $\bar{F}(\cdot)$ , respectively |
| $h(x)$         | The generalized failure rate function of $\varepsilon$ , i.e., $h(x) = xf(x)/\bar{F}(x)$   |
| $p$            | The retail price set by the distributor, a decision variable   |
| $D(p, \theta)$ | The market demand when the retail price is $p$ and the freshness level is $\theta$   |
| $y(t)$         | $\doteq y_0(\theta(t))$  |
| $k(t)$         | $\doteq k_0(\theta(t))$  |
| $c_1$          | Unit production cost of the producer   |
| $c_2$          | Unit transportation cost of the 3PL provider   |
| $s$            | Unit base transportation fee charged by the 3PL provider, a decision variable  |
| $\gamma(t)$    | The compensation factor w.r.t. the actual transportation time $t$  |
| $\gamma_0$     | Expected value of $\gamma(t)$ , w.r.t. the realized transportation time $t$ , i.e., $\gamma_0 = \mathbb{E}\{\gamma(T)\}$                 |
| $q$            | The producer's shipping quantity, a decision variable  |
| $w$            | The producer's wholesale price, a decision variable  |
| $\hat{q}$      | The distributor's purchasing quantity, a decision variable   |
| $\pi_r(\cdot)$ | The 3PL provider's expected profit function  |
| $\pi_m(\cdot)$ | The producer's expected profit function  |
| $\pi_d(\cdot)$ | The distributor's expected profit function   |
| $q_c$          | The shipping quantity in the centralized system  |
| $p_c$          | The retail price in the centralized system   |
| $\Pi_c(\cdot)$ | The expected profit of the entire chain in the centralized system  |



shortage cost. Fig. 1 provides a simple illustration of the model; and Table 1 lists the notation used in the paper.

To proceed, we summarize the sequence of events as follows. (1) The 3PL provider determines a base transportation price  $s$ , and offers a committed transportation time  $t_0$  and a compensation function  $\gamma(t)$  for  $t > t_0$ . (2) The producer determines the shipping quantity  $q$ , and the product is loaded onto the vessel. (3) The product arrives at the wholesale market after time  $t$ , and the transaction between the 3PL provider and the producer is settled. (4) The producer sets a wholesale price  $w$ . (5) The distributor determines the purchase quantity  $\hat{q}$  and the retail price  $p$ ; the transaction between the producer and the distributor is settled. (6) The end-customer demand is realized and satisfied by the distributor.

Before proceeding to study the optimal decisions, we formulate the decision problems for the three parties in the following.

- The 3PL provider determines a unit base transportation fee  $s$ , by taking into account the possible compensation to the producer. The expected profit function,  $\pi_\ell(\cdot)$ , is

$$\pi_\ell(s) = \mathbb{E}_T\{[s\gamma(T) - c_2]q\} = (s\gamma_0 - c_2)q, \tag{1}$$

where  $q$  is the shipping quantity of the producer, which is influenced by  $s$ .

- The producer determines the shipping quantity  $q$  before the product is transported, and the wholesale price  $w$  after the product arrives at the wholesale market. The expected profit,  $\pi_m(\cdot)$ , is

$$\pi_m(q, w) = \mathbb{E}_T\{w \min(\hat{q}, qm(T)) - [c_1 + s\gamma(T)]q\}, \tag{2}$$

where  $\hat{q}$  is the quantity that the distributor will purchase from the producer.

- Given the realized transportation time being  $t$  (and therefore the freshness level being  $\theta(t)$ ) and the wholesale price offered by the producer being  $w$ , the distributor determines the purchasing quantity  $\hat{q}$  and the retail price  $p$  to maximize his expected profit  $\pi_d(\cdot)$

$$\pi_d(p, \hat{q} | t) = p \cdot \mathbb{E}_\varepsilon\{\min(D(p, \theta(t)), \hat{q})\} - w\hat{q}. \tag{3}$$

#### 4. Optimal decentralized decisions

In this section we will investigate the optimal decisions for the 3PL provider, the producer, and the distributor in the decentralized supply chain. First, we will derive the optimal decisions of the distributor, given an arbitrary wholesale price  $w$  and freshness level  $\theta(t)$ . We will then derive the optimal wholesale price of the producer, given transportation time  $t$  and shipping quantity  $q$ . This will be followed by the determination of optimal shipping quantity  $q$ , given the 3PL provider's unit transportation fee  $s$ . And finally, we will study the 3PL provider's optimal pricing decision.

##### 4.1. Optimal decisions of the distributor

The distributor faces a joint quantity-pricing decision problem for any given freshness level  $\theta(t)$  and wholesale price  $w$ . For ease of exposition, we denote

$$k(t) := k_0(\theta(t)), \quad y(t) := y_0(\theta(t)),$$

and define the following "stocking factor" (see [32])

$$z := \hat{q}/[y(t)p^{-k(t)}]. \tag{4}$$

Then, the problem of optimizing  $(\hat{q}, p)$  can be converted into that of optimizing  $(\hat{q}, z)$ . By substituting (4) into (3), the distributor's

objective function can be rewritten as

$$\pi_d(z, \hat{q} | t) = (zy(t))^{1/k(t)} \hat{q}^{1-1/k(t)} \left[ 1 - \int_0^z (1-x/z)f(x) dx \right] - w\hat{q}. \tag{5}$$

**Theorem 1.** For any realized transportation time  $t$  and wholesale price  $w$ :

- (i) the optimal stocking factor  $z^*(t)$  is the unique solution to the following equation:

$$\int_0^z (k(t)-1)xf(x) dx = z[1-F(z)]. \tag{6}$$

- (ii) the optimal purchase quantity of the distributor is given by  $\hat{q}^*(t, w) = z^*(t)y(t)[(1-F(z^*(t)))/w]^{k(t)}$ .

From (6) we can see that the optimal stocking factor is determined by the price-elasticity  $k(t)$  and the distribution of the random factor  $\varepsilon$ , and is independent of other parameters. Taking derivative with respect to the transportation time  $t$  on both sides of (6) and by some algebraic transformations, we have

$$\frac{dz^*(t)}{dt} = \frac{k'_0(\theta)\theta'(t) \left[ \int_0^{z^*(t)} uf(u) du \right]^2}{\bar{F}(z^*(t)) \int_0^{z^*(t)} [h(u)-h(z^*(t))]\bar{F}(u) du}.$$

As  $h(u) \leq h(z^*(t))$  for any  $u \leq z^*(t)$  and  $\theta'(t) \leq 0$  (Assumption 1), the monotonicity of  $z^*(t)$  with respect to the transportation time  $t$  is the same as that of  $k_0(\theta)$  with respect to the freshness level  $\theta$ . As a result,  $z^*(t)$  is decreasing in  $t$ ; i.e., the optimal stocking factor is lower when the product has a higher price elasticity (i.e.,  $k(t)$  is larger). Note also that we assume the freshness index  $\theta(t)$  to be decreasing in time  $t$ ; therefore, the optimal stocking factor is higher when the product is fresher. This implies that the perishability of product may reduce the distributor's stocking factor. The distributor's optimal joint purchasing quantity and pricing decisions are summarized in the following Corollary.

**Corollary 1.** Suppose that the producer offers a wholesale price  $w$  and the realized transportation time is  $t$ . The distributor should purchase a quantity  $\hat{q}^* = \min(\hat{q}^*(t, w), qm(t))$ , and set the retail price at

$$p^* = \left[ \frac{z^*(t)y(t)}{\hat{q}^*} \right]^{1/k(t)}. \tag{8}$$

Note that if  $qm(t) < \hat{q}^*(t, w)$ , then the maximal quantity that the distributor can purchase from the producer is  $qm(t)$ , and we can intuitively see that the optimal retail price should be

$$p^* = \left[ \frac{z^*(t)y(t)}{qm(t)} \right]^{1/k(t)}.$$

When the producer has sufficient supply, i.e.,  $qm(t) \geq \hat{q}^*(t, w)$ , based on (7) and (8) we have

$$p^* = \frac{w}{\bar{F}(z^*(t))},$$

which implies that the distributor's optimal retail price  $p^*$  is proportional to the producer's wholesale price  $w$ . It is easy to see that  $p^* > w$ . This guarantees that the distributor always has a positive expected profit. The incremental price (as relative to the wholesale price) depends on the optimal stocking factor  $z^*(t)$ : the larger the  $z^*(t)$ , the higher the optimal retail price. From the monotonic relationship between  $z^*(t)$  and  $\theta(t)$ , we know that the distributor should set a higher retail price when the price-elasticity is lower and/or the product is fresher.

#### 4.2. Optimal decisions of the producer

The producer faces a two-stage decision problem. First, we investigate the optimal wholesale price in the second stage, provided that the initial shipping quantity is  $q$  and the realized transportation time is  $t$ . As the distributor will buy  $\hat{q}^*(t, w)$  units when the wholesale price is  $w$ , the eventual quantity transacted will be  $\min(qm(t), \hat{q}^*(t, w))$ . Because the production cost and transportation cost are both sunk, the producer seeks to maximize his selling revenue by setting an appropriate wholesale price. The objective function is

$$\pi_m(w|q, t) = w \min(qm(t), \hat{q}^*(t, w)). \tag{9}$$

**Theorem 2.** For any shipping quantity  $q$  and realized transportation time  $t$ , the producer should set his wholesale price at

$$w^*(t, q) = \left[ \frac{z^*(t)y(t)}{qm(t)} \right]^{1/k(t)} \bar{F}(z^*(t)). \tag{10}$$

As a result, the surviving quantity after the transportation,  $qm(t)$ , exactly matches the distributor's optimal purchase quantity  $\hat{q}^*(t, w)$ .

Theorem 2 implies that the optimal wholesale price should be set at the point such that the quantity to be purchased by the distributor exactly equals the producer's effective supply; i.e.,  $qm(t) = \hat{q}^*(t, w)$ . This may be attributed to the assumption that any product left unsold generates no salvage value, whereas the market demand is sensitive to the retail price, i.e.,  $k(t) = k_0(\theta(t)) > 1$ . Note that if  $k(t) \leq 1$ , we can show that the producer's revenue is also increasing in  $w$  when  $w \geq w^*(t, q)$ ; therefore, the optimal wholesale price will go to infinity.

It is interesting to analyze the relationship between the optimal wholesale price  $w^*(t, q)$  and the realized transportation time  $t$ . Intuitively, one might have expected that the producer should set a lower wholesale price if the realized transportation time is longer, because the product is less fresh. However, Eq. (10) shows that  $w^*(t, q)$  may not be decreasing in  $t$  because the realized transportation time affects not only the product's freshness, but also the optimal stocking factor  $z^*(t)$  and surviving factor  $m(t)$ . Consider an extreme case in which  $m(t)$  is very close to zero for a high value of  $t$ , when the optimal retail price as given by (10) could be very high. The reason is that the product supply is reduced.

Next we consider the optimal shipping quantity for the producer. Substituting (10) into (2) and letting  $A(t) = (z^*(t) y(t))^{1/k(t)} m(t)^{1-1/k(t)} \bar{F}(z^*(t))$ , the producer's expected profit can be expressed as

$$\begin{aligned} \pi_m(q) &= \mathbb{E}_t[\pi_m(w^*(t, q)|q, t)] - (c_1 + s\gamma_0)q \\ &= \mathbb{E}_t[w^*(t, q)qm(t)] - (c_1 + s\gamma_0)q \\ &= \mathbb{E}_t[(z^*(t)y(t))^{1/k(t)} m(t)^{1-1/k(t)} \bar{F}(z^*(t))q^{1-1/k(t)}] - (c_1 + s\gamma_0)q \\ &= \int_a^b A(t)q^{1-1/k(t)} g(t) dt - (c_1 + s\gamma_0)q. \end{aligned} \tag{11}$$

**Theorem 3.** Given the 3PL provider's unit base transportation fee is  $s$ , the producer's optimal shipping quantity  $q^*(s)$  is the unique solution of the following equation:

$$\int_a^b \left(1 - \frac{1}{k(t)}\right) A(t)q^{-1/k(t)} g(t) dt = c_1 + s\gamma_0. \tag{12}$$

It is obvious from (12) that  $q^*(s)$  is decreasing in  $s$ . This is consistent with the general intuition: if the 3PL provider charges a higher unit transportation fee, the producer will ship a smaller quantity of the product. One might be interested in how the product perishability influences the producer's shipping decision. For example, since a portion of the product might become

unsaleable, one may expect the producer to ship more product to account for such quantity loss. However, from (12) it is not difficult to show that the existence of function  $m(t) \in (0, 1)$  actually reduces the optimal shipping quantity  $q^*(s)$  (assuming all other parameters remain unchanged). This is because the potential quantity loss means a higher unit effective cost for the producer, and consequently the shipping quantity should be lowered.

#### 4.3. Optimal decision of the 3PL provider

Having obtained the relationship between the producer's shipping quantity  $q^*(s)$  and the 3PL provider's unit transportation fee  $s$ , we are ready to study the 3PL provider's optimal pricing decision. Our main result is summarized in the following theorem.

**Theorem 4.** The 3PL provider's optimal transportation fee  $s^*$  is

$$s^* = \frac{1}{\gamma_0} \left[ c_2 + \int_a^b \left(1 - \frac{1}{k(t)}\right) \frac{1}{k(t)} A(t)(q^*)^{-1/k(t)} g(t) dt \right], \tag{13}$$

where  $q^*$  is the unique solution of

$$\int_a^b \left(1 - \frac{1}{k(t)}\right)^2 A(t)(q^*)^{-1/k(t)} g(t) dt = c_1 + c_2. \tag{14}$$

Note that (14) reveals the producer's optimal shipping quantity. It follows from (13) that  $s^*$  is decreasing in  $\gamma_0$ , which implies that the 3PL provider should increase the transportation fee if it offers a higher compensation rate. For the case with  $k_0(\theta) = k$  (i.e., the price elasticity is independent of the freshness level  $\theta$ ), we can obtain a closed-form formulation for  $s^*$

$$s^* = \frac{1}{\gamma_0} \left[ c_2 + \frac{c_2 + c_1}{k-1} \right]. \tag{15}$$

Eq. (15) shows that when the price-elasticity is a constant,  $s^*$  is decreasing in  $k$ , which implies that the 3PL provider should reduce his transportation fee if the downstream market demand is more sensitive to the retail price.

Note that we consider the transportation fee as an endogenous decision made by the 3PL provider who acts as a Stackelberg-game leader. In a perfectly competitive market, however, it is often determined exogenously. Even in such situations, the 3PL provider can still gain certain advantages by providing appropriate compensation schemes to the clients. For example, suppose that the unit transportation fee charged by the 3PL provider as a function of the realized delivery time  $t$  is

$$s(t) = s_0 - \eta(t - t_0)^+,$$

where  $s_0$  is the exogenous base transportation fee,  $t_0$  is the committed delivery time and  $\eta$  is the compensation rate per unit time if transportation delay occurs. Using a similar procedure as above, the 3PL provider can optimize his optimal committed delivery time and/or compensation rate by considering their impact on the producer's shipping quantity.

#### 4.4. The case with a constant price elasticity

As we can see from the above subsections, most of the optimal decisions of the supply chain players do not have a closed-form. This hinders us from uncovering more managerial insights. In this subsection we discuss a special case in which the price-elasticity is independent of the freshness level. That is, the demand function is reduced to the form  $D(p, \theta) = y_0(\theta)p^{-k}\epsilon$ . As will be shown, the optimal decisions can be characterized in an explicit form in this case.

Firstly, it follows from (6) that  $z^*(t)$  becomes independent of the realized transportation time  $t$  when the price elasticity becomes a constant. For simplicity, we denote the optimal stocking factor as  $z_0$ . Our main results are summarized in the following Corollary.

**Corollary 2.** Suppose that the price-elasticity is a constant  $k$ , and let  $B_0 = \mathbb{E}_T\{y(T)^{1/k}m(T)^{1-1/k}\}$ .

(i) The 3PL provider's optimal transportation fee  $s^*$  is given by (15), and the corresponding optimal expected profit is

$$\pi_\ell^* = \frac{c_1 + c_2}{k-1} z_0 \left[ \left( \frac{k-1}{k} \right)^2 \times B_0 \frac{1-F(z_0)}{c_1 + c_2} \right]^k.$$

(ii) The producer's optimal shipping quantity is

$$q^* = z_0 \left[ \left( \frac{k-1}{k} \right)^2 \times B_0 \frac{1-F(z_0)}{c_1 + c_2} \right]^k.$$

The optimal wholesale price corresponding to a realized transportation time  $t$  is

$$w^* = \frac{(c_1 + c_2)k^2}{B_0(k-1)^2} \left[ \frac{y(t)}{m(t)} \right]^{1/k}.$$

Accordingly, the producer's expected profit is

$$\pi_m^* = \frac{k(c_1 + c_2)}{(k-1)^2} z_0 \left[ \left( \frac{k-1}{k} \right)^2 \times B_0 \frac{1-F(z_0)}{c_1 + c_2} \right]^k.$$

(iii) For any realized transportation time  $t$ , the distributor's optimal purchasing quantity and retail price are

$$\hat{q}^* = q^*m(t) = z_0m(t) \left[ \left( \frac{k-1}{k} \right)^2 \times B_0 \frac{1-F(z_0)}{c_1 + c_2} \right]^k,$$

$$p^* = \frac{(c_1 + c_2)k^2}{B_0(k-1)^2[1-F(z_0)]} \left[ \frac{y(t)}{m(t)} \right]^{1/k}.$$

The distributor's expected profit is

$$\pi_d^* = \frac{k^2(c_1 + c_2)}{(k-1)^3} z_0 \left[ \left( \frac{k-1}{k} \right)^2 \times B_0 \frac{1-F(z_0)}{c_1 + c_2} \right]^k.$$

From Corollary 2, we can see that the relative profits of the three supply chain members are

$$\pi_\ell^* : \pi_m^* : \pi_d^* = 1 : \frac{k}{k-1} : \left( \frac{k}{k-1} \right)^2.$$

Since the price-elasticity is greater than 1,  $\pi_\ell^* < \pi_m^* < \pi_d^*$  and the above ratio depends only on  $k$ . The result above shows that, without coordination, the downstream distributor achieves the largest portion of the profit of the entire supply chain, whereas the upstream 3PL provider gains the smallest, especially when the price elasticity in the destination market is small.

## 5. Fully centralized and partially centralized decisions

By a "fully centralized" supply chain, we mean that the 3PL provider, the producer, and the distributor act in a coordinated way under a joint objective of maximizing the expected profit of the entire supply chain. The optimal decisions in this setting are useful when the three firms belong to a single organization that seeks to optimize its global objective. In practice, it is also possible that only two of the three firms coordinate to maximize their joint objective. In such cases, we say the supply chain "partially centralized". In this section we will derive the optimal

decisions for the fully centralized and partially centralized supply chains, respectively.

### 5.1. Optimal fully centralized decisions

In a fully centralized supply chain, the transportation fee, the wholesale price, and the distributor's purchase quantity all become internal parameters. There are only two decisions that need to be made, which are the shipping quantity (denoted as  $q_c$ ) and the retail price (denoted as  $p_c$ ).

The expected profit function, denoted as  $\Pi_c$ , becomes

$$\Pi_c(q_c) = \mathbb{E}_t\{\Pi_c(p_c|q_c, t)\} - q_c(c_2 + c_1), \quad (16)$$

where

$$\Pi_c(p_c|q_c, t) = p_c \mathbb{E}_\varepsilon\{\min(q_c m(t), D(p_c, t))\}. \quad (17)$$

The optimal decisions can also be derived in a backward order; the results are summarized in the theorem.

**Theorem 5.** In the fully centralized system:

(i) Given any shipping quantity  $q_c$  and realized transportation time  $t$ , the optimal retail price is

$$p_c^* = \left[ \frac{z^*(t)y(t)}{q_c m(t)} \right]^{1/k(t)}, \quad (18)$$

where  $z^*(t)$  satisfies Eq. (6).

(ii) The optimal shipping quantity is uniquely determined by the following equation:

$$\int_a^b A(t)q_c^{-1/k(t)}g(t)dt = c_1 + c_2. \quad (19)$$

To compare the optimal decisions and performances of the decentralized and the fully centralized supply chains, we first present the following theorem.

**Theorem 6.** The optimal shipping quantity in the fully centralized system is greater than that in the decentralized system, i.e.,  $q_c^* > q^*$ .

For the constant price-elasticity case with  $k_0(\theta) = k$ , we can easily have

$$q_c^* : q^* = \left( \frac{k}{k-1} \right)^{2k} > 1,$$

which indicates that the ratio  $q_c^* : q^*$  depends only on the price elasticity of the market demand. It is not difficult to see that  $[k/(k-1)]^{2k}$  is decreasing in  $k \in (1, \infty)$ . Therefore, the more sensitive the market demand to the retail price (i.e., the larger the  $k$  value), the closer the optimal shipping quantity in the fully centralized system to that in the decentralized system.

Next, we study the effects of coordination on the expected profit. Recall that the 3PL provider, the producer and the distributor's expected profits in the decentralized system are  $\pi_\ell^*$ ,  $\pi_m^*$ , and  $\pi_d^*$ , respectively. We define  $(\pi_\ell^* + \pi_m^* + \pi_d^*)$  as the system profit in the absence of coordination, and we are interested in the magnitude of the expected loss due to lack of coordination among the supply chain members. For the case with constant price elasticity, we have

$$\varsigma = 1 - \frac{\pi_\ell^* + \pi_m^* + \pi_d^*}{\Pi_c^*} = 1 - \left( \frac{k-1}{k} \right)^{2k} - \left( \frac{k-1}{k} \right)^{2k-1} - \left( \frac{k-1}{k} \right)^{2k-2}.$$

It can be shown that the relative profit loss,  $\varsigma$ , is increasing in  $k$  (see curve (a) in Fig. 2). That is, the more sensitive the market demand to a change in price, the larger the profit loss due to lack of coordination.

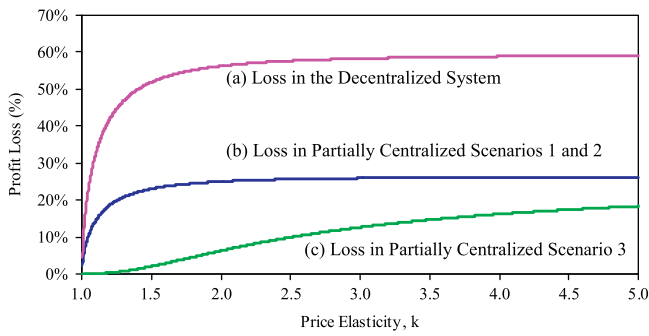


Fig. 2. Losses compared to the centralized system for constant elasticity.

5.2. The partially centralized decisions

Since the supply chain under consideration consists of three firms, there are three possible partially centralized systems, which we call as Scenarios 1, 2, and 3:

*Scenario 1* The 3PL provider and the producer seek to optimize a joint objective. This is equivalent to the case in which the producer ships the product through his own transportation fleet. In this scenario, the producer determines his shipping quantity (before transportation) and wholesale price (after transportation), and then the distributor makes his decisions on the purchasing quantity and retail price.

*Scenario 2* The producer and the distributor seek to optimize a joint objective. This is equivalent to the case in which the producer ships the product through logistics outsourcing, and sells them directly to the distant retail market. In this scenario, the 3PL provider first determines his unit transportation fee, and then the producer/distributor determines his shipping quantity (before transportation) and retail price (after transportation).

*Scenario 3* The 3PL provider and the distributor seek to optimize a joint objective. This is equivalent to the case in which the producer ships the product through the downstream distributor’s transportation fleet. In this scenario, the 3PL provider first determines his unit transportation fee; the producer determines his shipping quantity (before transportation) and wholesale price (after transportation); and then the distributor makes his decisions on the purchasing quantity and retail price.

These partially centralized supply chains are actually two-echelon systems with two independent profit centers. Following a similar backward deduction procedure as used in Section 4, we can derive the optimal decisions for the three scenarios; major results are presented in the following theorem.

**Theorem 7.** *The optimal decisions in the partially centralized scenarios are as follows:*

(a) *In scenario 1, the producer’s optimal shipping quantity  $q_1^*$  is the unique solution of the following equation:*

$$\int_a^b \left(1 - \frac{1}{k(t)}\right) A(t)q^{-1/k(t)}g(t) dt = c_1 + c_2.$$

*Given a realized transportation time  $t$ , the producer should set his wholesale price at*

$$w_1^*(t) = \left[\frac{z^*(t)y(t)}{q_1^*m(t)}\right]^{1/k(t)} \bar{F}(z^*(t)).$$

*The distributor’s optimal purchasing quantity is  $q_1^*m(t)$  and the retail price is*

$$p_1^* = \left[\frac{z^*(t)y(t)}{q_1^*m(t)}\right]^{1/k(t)}.$$

(b) *In scenario 2, the 3PL provider’s optimal transportation fee,  $s_2^*$ , is*

$$s_2^* = \frac{1}{\gamma_0} \left[ \int_a^b A(t)(q_2^*)^{-1/k(t)}g(t) dt - c_1 \right],$$

*where  $q_2^* = q_1^*$ , which is the producer’s optimal shipping quantity. Given a realized transportation time  $t$ , the distributor (producer) should set his retail price at  $p_2^* = p_1^*$ .*

(c) *In scenario 3, the 3PL provider’s optimal transportation fee,  $s_3^*$ , is*

$$s_3^* = \frac{1}{\gamma_0} \left[ \int_a^b \left(1 - \frac{1}{k(t)}\right) A(t)(q_3^*)^{-1/k(t)}g(t) dt - c_1 \right],$$

*where  $q_3^*$  is the optimal shipping quantity of the producer, the unique solution of the following equation:*

$$\int_a^b \frac{k^2(t) - k(t) + 1}{k^2(t)} A(t)q^{-1/k(t)}g(t) dt = c_1 + c_2.$$

*Given a realized transportation time  $t$ , the producer should set his wholesale price at*

$$w_3^*(t) = \left[\frac{z^*(t)y(t)}{q_3^*m(t)}\right]^{1/k(t)} \bar{F}(z^*(t)),$$

*and the distributor’s optimal purchase quantity is  $q_3^*m(t)$  and retail price is*

$$p_3^* = \left[\frac{z^*(t)y(t)}{q_3^*m(t)}\right]^{1/k(t)}.$$

A very interesting finding can be observed from Theorem 7. From part (c), we can show that the unit expected transportation profit,  $s_3^*\gamma_0 - c_2$ , is equal to

$$s_3^*\gamma_0 - c_2 = - \int_a^b \frac{1}{k^2(t)} A(t)(q_3^*)^{-1/k(t)}g(t) dt,$$

which is negative. This implies that to optimize their joint profit, the 3PL provider and the distributor may even choose to sacrifice their transport-related profit. This is equivalent to a situation in which an alliance compensates the producer for the transportation cost to motivate him to increase the shipping quantity. As a result, compared to the fully decentralized system, both the producer and the alliance of the 3PL provider and the distributor will be better off.

From Theorem 7, we can easily have the following relationship:

$$q_c^* > q_3^* > q_2^* = q_1^* > q^*;$$

that is, the optimal shipping quantities of the three partially-centralized systems lie between those of the decentralized and fully-centralized systems. As one might expect, the system-wide performance of the partially-centralized systems out-performs that of the decentralized system, but is lower than that of the fully-centralized system. Let us consider the case with a constant price-elasticity; the optimal decisions and corresponding profits in the three scenarios are summarized in Table 2.

We have shown that, compared to the fully centralized system, the loss in the decentralized supply chain could be rather high, especially when the market is very price-sensitive. However, as shown by curves (b) and (c) in Fig. 2, if the 3PL provider collaborates with the producer (scenario 1), or if the producer collaborates with the distributor (scenario 2), then the loss in the



**Table 2**  
Summary of optimal decisions and profits in different scenarios.

|                                  | Partially-centralized systems  |  |   | Decentralized system   |
|----------------------------------|--|--|---|--|
|                                  | Scenario 1   | Scenario 2   | Scenario 3  |  |
| Transportation Fee, $s^*$        | N/A  | $\frac{1}{\gamma_0} [c_2 + \frac{c_1 + c_2}{k-1}]$ | $\frac{1}{\gamma_0} \left[ \frac{k^2 - k}{k^2 - k + 1} (c_1 + c_2) - c_1 \right]$ | $\frac{1}{\gamma_0} [c_2 + \frac{c_1 + c_2}{k-1}]$                         |
| Wholesale price, $w^*$           | $\frac{(c_1 + c_2)k}{B_0(k-1)} \left( \frac{y(t)}{m(t)} \right)^{1/k}$ | N/A  | $\frac{(c_1 + c_2)k^2}{B_0(k^2 - k + 1)} \left( \frac{y(t)}{m(t)} \right)^{1/k}$  | $\frac{(c_1 + c_2)k^2}{B_0(k-1)^2} \left( \frac{y(t)}{m(t)} \right)^{1/k}$ |
| Shipping quantity, $q^*$         | $\left( \frac{k-1}{k} \right)^k q_c^*$                                 | $\left( \frac{k-1}{k} \right)^k q_c^*$             | $\left( \frac{k^2 - k + 1}{k^2} \right)^k q_c^*$                                  | $\left( \frac{k-1}{k} \right)^{2k} q_c^*$                                  |
| 3PL provider's profit, $\pi_l^*$ | $\left( \frac{k-1}{k} \right)^k \Pi_c^*$                               | $\left( \frac{k-1}{k} \right)^k \Pi_c^*$           | $\left[ \left( \frac{k^2 - k + 1}{k^2} \right)^k \Pi_c^* \right]^a$               | $\left( \frac{k-1}{k} \right)^{2k} \Pi_c^*$                                |
| Producer's profit, $\pi_m^*$     |  | $\left( \frac{k-1}{k} \right)^{k-1} \Pi_c^*$       | $\frac{(k-1)(k^2 - k + 1)^{k-1}}{k^{2k-1}} \Pi_c^*$                               | $\left( \frac{k-1}{k} \right)^{2k-1} \Pi_c^*$                              |
| Distributor's profit, $\pi_d^*$  | $\left( \frac{k-1}{k} \right)^{k-1} \Pi_c^*$                           |  | $\left[ \left( \frac{k^2 - k + 1}{k^2} \right)^k \Pi_c^* \right]^a$               | $\left( \frac{k-1}{k} \right)^{2k-2} \Pi_c^*$                              |

<sup>a</sup> The joint profit of the 3PL provider and the distributor.

entire supply chain could be decreased significantly. If the 3PL provider collaborates with the distributor, then the system-wide profit could be enhanced to the maximal extent, because by decreasing the transportation fee the alliance of the 3PL provider and the distributor could motivate the producer to choose a shipping quantity that is closer to  $q_c^*$ . Fig. 2 can also be read from another perspective: As compared to the decentralized system, a partially centralized system (especially Scenario 3) can significantly improve the performance of the entire supply chain. This is because the alliance between two of the three parties can remove one of the two sources of “double marginalization” that exist in the three-tier supply chain.

While the three partially centralized systems generate many interesting phenomena (which have apparently not been addressed in the literature), partial coordination is inefficient from the perspective of optimizing the entire supply chain. The ideal scenario is, of course, that the three parties coordinate so that the maximum profit of the fully centralized system is achieved. This will be investigated in the next section.

## 6. Design of coordination mechanism

As it is well known, the key of a coordination mechanism (such as buy-back, quantity discount, etc.) is to motivate the downstream distributor (or retailer) to order up to the same level as that in the centralized supply chain. For this purpose, the upstream supplier usually offers to bear a portion of the distributor's risk. In the business model of our paper, however, the producer faces the risk that the product may decay/deteriorate before reaching the wholesale market. The shipping quantity, which is one of the major factors that affect the performance of the entire supply chain, is determined by the upstream producer. Moreover, the distributor faces an uncertain market demand, and the 3PL provider faces a cost penalty if there is a transportation delay. Therefore, a mechanism that coordinates the three parties should be able to share their respective risks.

We propose a “wholesale market clearance (WMC)” contract between the producer and the distributor, under which the distributor agrees to purchase all the surviving products through a specified wholesale pricing criteria. As we will show, this WMC contract shares some similarities to the traditional quantity-

discount contract. Meanwhile, we propose to adopt another contract, the wholesale-price discount sharing (WDS), between the 3PL provider and the producer. Implementing of the two contracts depends on the realized transportation time  $t$ .

The set of WMC and WDS contracts runs in a sequential order. First, the producer negotiates with the distributor and a wholesale pricing policy is determined. Then the producer negotiates with the 3PL provider to determine the transportation fee charging policy. The second negotiation may depend on the result of the first one. In both contracts, the producer promises to ship the quantity  $q = q_c$ , where  $q_c$  is the optimal shipping quantity of the fully centralized system. Specifically:

(1) Our WMC contract suggests that the distributors agree to purchase all the surviving quantities  $q m(t)$ , at a wholesale price as follows:

$$w(q, t) = (1 - \alpha) \left[ \frac{z^*(t)y(t)}{qm(t)} \right]^{1/k(t)} \mathbb{E}_\varepsilon \left\{ \min \left( 1, \frac{\varepsilon}{z^*(t)} \right) \right\} + \alpha \frac{c_1 + c_2}{m(t)}, \quad (20)$$

where  $q = q_c$  is the producer's shipping quantity,  $t$  is the realized transportation time, and  $\alpha$  is a constant within (0,1) that is to be determined in the negotiation between the producer and the distributor. Note that for any  $t$ ,  $w(q, t)$  is strictly decreasing in the surviving quantity  $q m(t)$ . This is similar to the traditional quantity discount contract.

The intuition behind the WMC contract can be interpreted in another way as follows. By multiplying  $qm(t)$  on both sides of Eq. (20), we have

$$w(q, t)qm(t) = (1 - \alpha) [z^*(t)y(t)]^{1/k(t)} (qm(t))^{1-1/k(t)} \mathbb{E}_\varepsilon \left\{ \min \left( 1, \frac{\varepsilon}{z^*(t)} \right) \right\} + \alpha(c_1 + c_2)q. \quad (21)$$

According to Eq. (5), we know that

$$SR := [z^*(t)y(t)]^{1/k(t)} (qm(t))^{1-1/k(t)} \mathbb{E}_\varepsilon \left\{ \min \left( 1, \frac{\varepsilon}{z^*(t)} \right) \right\}$$

represents the distributor's selling revenue (SR), where  $qm(t)$  is the available quantity to be sold to end customers. Therefore, (21) is equivalent to

$$w(q, t)qm(t) - (c_1 + c_2)q = (1 - \alpha)[SR - (c_1 + c_2)q]. \quad (22)$$

Clearly, the LHS of (22) is the joint profit of the producer and the 3PL provider, and  $(SR - (c_1 + c_2)q)$  in the RHS is the total profit of

the fully centralized system. Therefore, (22) implies, by the WMC contract, the shares of profit of the producer/3PL and the distributor are  $(1-\alpha)$  and  $\alpha$ , respectively. The distributor promises to purchase all the surviving quantities  $qm(t)$ . The further division of the profit between the producer and the 3PL will be a result of negotiation between them.

(2) Our WDS contract suggests that the transportation fee to be charged by the 3PL provider should take the following form:

$$s(q,t) = c_2 + \beta[m(t)w(q,t) - (c_1 + c_2)]. \tag{23}$$

That is, the transportation fee depends on the realized transportation time  $t$  and the producer's actual wholesale price. The parameter  $\beta$  is a constant in  $(0,1)$ , which is to be determined in the negotiation process between the producer and the 3PL provider.

We can show that (23) is equivalent to the following form:

$$s(q,t) - s_0(t) = \beta m(t)[w(q,t) - w_0],$$

where  $w_0$  is the list wholesale price, and  $s_0(t)$  can be regarded as a base transportation fee that is given by

$$s_0(t) = c_2 - \beta(c_1 + c_2) + \beta m(t)w_0.$$

Relationship (23) indicates that the 3PL provider should offer a transportation-fee discount that is dependent upon the producer's wholesale-price discount. Note that the producer will pay a lower transportation fee if he has to reduce his wholesale price. As a result, the 3PL provider shares a portion of the cost that the producer incurs from his wholesale-price discount. For additional discussion on the price-discount sharing contract, see Bernstein and Federgruen [5], or Li and Atkins [26]. By substituting (20) into (23), we have

$$s(q,t) = c_2 + \beta(1-\alpha) \left[ m(t) \left( \frac{z^*(t)y(t)}{qm(t)} \right)^{1/k(t)} \mathbb{E}_\varepsilon \left\{ \min \left( 1, \frac{\varepsilon}{z^*(t)} \right) \right\} - (c_1 + c_2) \right]. \tag{24}$$

**Proposition 1.**  $s(q,t)$  is decreasing in the realized transportation time  $t$ .

Proposition 1 suggests that the 3PL provider should decrease his unit transportation fee when there is a transportation delay. This is incentive compatible. Moreover,  $s(q,t)$  is strictly decreasing in the producer's shipping quantity,  $q$ , meaning that for any realized transportation time a lower transportation fee is charged for a larger shipping quantity. This, again, shares the concept of quantity discount (or price discount) and is incentive compatible.

**Theorem 8.** The WMC contract (20), together with the WDS contract (23), will induce the decentralized supply chain to achieve the same performance as that of the centralized supply chain for any  $\alpha, \beta \in (0,1)$ .

Moreover, it follows directly that, by adopting the WMC and WDS contracts, the producer's optimal expected profit is

$$\pi'_m = \pi'_m(q_c^*) = (1-\beta)(1-\alpha)\Pi_c(q_c^*) = (1-\beta)(1-\alpha)\Pi_c^*.$$

The distributor's optimal expected profit is

$$\pi'_d = \mathbb{E}_t \{ \pi'_d(z^*(t) | q_c^*, t) \} = \alpha \Pi_c(q_c^*) = \alpha \Pi_c^*.$$

The 3PL provider's optimal expected profit is

$$\pi'_\ell = \mathbb{E}_t \{ (s(q,t) - c_2) q_c^* \} = \beta(1-\alpha)\Pi_c^*.$$

Therefore, the profit shares of the 3PL provider, the producer, and the distributor, are  $\beta(1-\alpha)$ ,  $(1-\beta)(1-\alpha)$ , and  $\alpha$ , respectively; i.e., the value of  $\alpha$  and  $\beta$ , which is generally determined by the relative bargaining powers of the supply chain members, directly determines their respective profits. However, to ensure that every party is willing to participate in the coordination, the following

two conditions should be satisfied: (i) each party can achieve a higher profit than that in the decentralized supply chain; (ii) any two of them will not establish a coalition, i.e., the sum of their profits should be greater than that in the partially centralized systems.

Consider the case of constant price-elasticity with  $k_0(\theta) = k$ . To ensure coordination, the parameters  $\alpha$  and  $\beta$  should satisfy the following conditions:

$$\beta(1-\alpha) + (1-\beta)(1-\alpha) \geq \left( \frac{k-1}{k} \right)^k,$$

$$(1-\beta)(1-\alpha) + \alpha \geq \left( \frac{k-1}{k} \right)^{k-1},$$

$$\alpha + \beta(1-\alpha) \geq \left( \frac{k^2 - k + 1}{k} \right)^k,$$

$$\alpha \geq \left( \frac{k-1}{k} \right)^{2k-2},$$

$$(1-\beta)(1-\alpha) \geq \left( \frac{k-1}{k} \right)^{2k-1},$$

$$\beta(1-\alpha) \geq \left( \frac{k-1}{k} \right)^{2k}.$$

It is not difficult to see that there exist an upper and a lower bound on  $\alpha$  and  $\beta$ , as follows (also see Fig. 3):

$$\left( \frac{k-1}{k} \right)^{2k-2} \leq \alpha \leq 1 - \left( \frac{k-1}{k} \right)^k,$$

$$\frac{1}{1-\alpha} \max \left\{ \left( \frac{k^2 - k + 1}{k} \right)^k, \left( \frac{k-1}{k} \right)^{2k} \right\} \leq \beta \leq \min \left\{ \frac{1}{1-\alpha} \left( 1 - \left( \frac{k-1}{k} \right)^{k-1} \right), 1 - \frac{1}{1-\alpha} \left( \frac{k-1}{k} \right)^{2k-1} \right\}.$$

As we can see from Fig. 3, the value of  $\beta$  that is acceptable to both the 3PL provider and the producer depends on the value of  $\alpha$ : as  $\alpha$  grows, the acceptable range of  $\beta$  becomes narrower. This occurs because the shareable profit,  $(1-\alpha)\Pi_c^*$ , shrinks.

Finally, we remark that the buy-back agreement (or compensation contract), which is usually complementary to the price-discount sharing contract (see [5]), is not needed in our WDS contract. This makes the contract much simpler to implement. In fact, the implementation of the contracts follows two steps only: (i) after the product arrives at the destination market, a wholesale price is determined according to the WMC contract and the transaction between the producer and the distributor is settled;

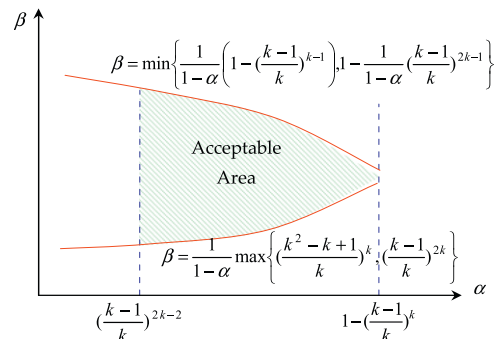


Fig. 3. The lower and upper bounds of  $\alpha$  and  $\beta$ .

and (ii) based on the wholesale price, the transportation fee is determined and the transaction between the producer and the 3PL provider is settled.

## 7. Concluding remarks

The main contributions of this paper can be summarized as follows:

- (1) A new model is established to address the supply chain management problem of a fresh product that involves a long distance transportation. Both types of perishability, deterioration and obsolescence, are considered. A multiplicative demand function is formulated, where the market size and the price elasticity are assumed to be functions of the product freshness to capture the sensitivity of the market demand to the product's freshness.
- (2) The optimal shipping quantity and the optimal wholesale price of the producer, the optimal transportation price of the 3PL provider, and the optimal purchasing quantity and retail price of the distributor are characterized and evaluated in a decentralized system (in which every party is an independent profit seeker), a fully centralized system (in which all three parties act to maximize their joint total profit), and partially centralized systems (in which two parties act together to maximize their joint objective but the other party acts independently).
- (3) An incentive scheme is developed to facilitate the coordination of the three parties, which include a wholesale-market clearance (WMC) contract between the producer and the distributor, and a wholesale-price-discount sharing (WDS) contract between the producer and the 3PL provider. We show that the proposed contracts allow the supply chain members to share the respective risks involved in the transportation and selling process, and eliminate the two sources of "double marginalization" that exist in the decentralized system.

Supply chains involving long distance transportation of fresh product have become increasingly common in both international and domestic markets, but investigation of such supply chains when transportation time is uncertain is a relatively new line of research. There are many interesting yet challenging issues left for future study. One topic is to further study the decisions faced by 3PL providers in different situations, such as cargo consolidation among multiple clients. Another topic is on the problem where the fresh product can be categorized into different freshness levels, and the distributor can set different retail prices accordingly. This problem is much more complicated, due to the correlation among the demands at different freshness levels. It is also interesting to consider the incomplete information issue. Note that our current model assumes that all the three parties have common knowledge on information such as market demand and cost of every party. This may not be a realistic assumption in many situations, although it is a common hypothesis made in the supply chain management literature. How to motivate all parties concerned to exchange information under certain incentive mechanisms? This is an interesting topic for future research. Other topics include multiple producers and multiple distributors, with or without cooperation.

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## Appendix

**Proof of Theorem 1.** (i) The optimal stocking factor that maximizes  $\pi_d(z, \hat{q} | t)$  must satisfy the following first-order condition:

$$\frac{\partial \pi_d(z, \hat{q} | t)}{\partial z} = \frac{[y(t)]^{1/k(t)} \hat{q}^{1-1/k(t)}}{z^{1-1/k(t)} k(t)} \left( 1 - \int_0^z \left[ \frac{x}{z} (k(t)-1) + 1 \right] f(x) dx \right) = 0,$$

from which we can show that optimal stocking factor  $z^*(t)$  must satisfy Eq. (6). We next prove the uniqueness of  $z^*(t)$ . Let

$$\phi(z) := \int_0^z [x(k(t)-1) + z] f(x) dx - z = -z\bar{F}(z) + (k(t)-1) \int_0^z x f(x) dx,$$

where  $\bar{F}(z) := 1 - F(z)$ . Then we have

$$\phi'(z) = zk(t)f(z) - \bar{F}(z) = k(t)\bar{F}(z) \left[ h(z) - \frac{1}{k(t)} \right],$$

where  $h(x)$  is the generalized failure rate function of  $\varepsilon$ , i.e.,  $h(x) = xf(x)/\bar{F}(x)$ . When  $h(x)$  increases in  $x$ , we know that  $\phi(z)$  decreases before  $z$  reaches  $h^{-1}(1/k(t))$  and increases after  $h^{-1}(1/k(t))$ , and hence is unimodal. As  $\phi(0) = 0$  and  $\lim_{z \rightarrow \infty} \phi(z) > 0$ , it is apparent that  $\phi(z) = 0$  has only one solution within  $(0, \infty)$ ; therefore,  $z^*(t)$  is uniquely determined by (6). It is also clear that for  $z > z^*(t)$ ,  $\phi(z) > 0$  and thus  $\partial \pi_d(z, \hat{q} | t) / \partial z < 0$ ; for  $0 < z < z^*(t)$ ,  $\phi(z) < 0$  and thus  $\partial \pi_d(z, \hat{q} | t) / \partial z > 0$ . Therefore,  $\pi_d(z, \hat{q} | t)$  is also unimodal in  $z$ , and  $z^*(t)$  is the unique maximizer of  $\pi_d(z, \hat{q} | t)$ .

(ii) It follows from (6) that the optimal stocking factor is independent of the other decision variable  $\hat{q}$ . Taking the first and second derivatives of  $\pi_d(z^*(t), \hat{q} | t)$  with respect to  $\hat{q}$ , we have

$$\frac{\partial \pi_d(z^*(t), \hat{q} | t)}{\partial \hat{q}} = (1 - F(z^*(t))) [z^*(t)y(t)]^{1/k(t)} \hat{q}^{-1/k(t)} - w,$$

$$\frac{\partial^2 \pi_d(z^*(t), \hat{q} | t)}{\partial \hat{q}^2} = -\frac{1 - F(z^*(t))}{k(t)} [z^*(t)y(t)]^{1/k(t)} \hat{q}^{-1/k(t)-1} < 0.$$

Therefore,  $\pi_d(z^*(t), \hat{q} | t)$  is strictly concave in  $\hat{q}$ , and the optimal  $\hat{q}^*$  that maximizes  $\pi_d(z^*(t), \hat{q} | t)$  is determined by the first-order condition, from which we have (7). This completes the proof.  $\square$

**Proof of Theorem 2.** We investigate the following two cases.

- (i) If  $qm(t) \leq \hat{q}^*(t, w)$ , i.e.,  $w \leq w^*(t, q)$ , then the maximal quantity that the distributor can purchase from the producer is  $qm(t)$ . As a result, the producer's revenue is  $wqm(t)$ , which is an increasing function of  $w$ . Therefore, the producer should set the wholesale price as high as possible (but of course, not greater than  $w^*(t, q)$ ).
- (ii) If  $qm(t) \geq \hat{q}^*(t, w)$ , i.e.,  $w \geq w^*(t, q)$ , then the final transacted quantity will be  $\hat{q}^*(t, w)$ , and the producer's revenue will be

$wq^* = z^*(t)y(t)\bar{F}(z^*(t))^{k(t)}w^{1-k(t)}$ , which is a decreasing function of  $w$  (recall that  $k_0(\theta) > 1$  for any freshness level  $\theta$ ). Therefore, the producer should set the wholesale price as low as possible (but not less than  $w^*(t, q)$ ).

Therefore, the producer’s revenue is a unimodal function *w.r.t.* the wholesale price; the maximal revenue is obtained at  $w = w^*(t, q)$ ; at this point the surviving quantity  $qm(t)$  exactly equals the distributor’s optimal purchase quantity  $\hat{q}^*(t, w)$ . This completes the proof.  $\square$

**Proof of Theorem 3.** Taking the first and second derivatives of  $\pi_m(q)$  *w.r.t.*  $q$ , we have

$$\frac{d\pi_m(q)}{dq} = \int_a^b \left(1 - \frac{1}{k(t)}\right) A(t)q^{-1/k(t)}g(t) dt - (c_1 + s\gamma_0),$$

$$\frac{d^2\pi_m(q)}{dq^2} = - \int_a^b \left(1 - \frac{1}{k(t)}\right) \frac{1}{k(t)} A(t)q^{-1-1/k(t)}g(t) dt < 0.$$

Therefore,  $\pi_m(q)$  is strictly concave in  $q$  because  $k(t) > 1$ . As a result, the optimal shipping quantity  $q^*(s)$  is uniquely determined by the first-order condition (12). This completes the proof.  $\square$

**Proof of Theorem 4.** From Theorem 3 we know that the producer’s optimal shipping quantity  $q^*(s)$  has a one-for-one mapping to  $s$ , which is given by (12). Therefore, to optimize  $s$  and thus maximize  $\pi_\ell(s)$ , the 3PL provider can choose a shipping quantity  $q^*(s)$  to maximize

$$\begin{aligned} \pi_\ell(s) &= (s\gamma_0 - c_2)q^*(s) \\ &= \left[ \int_a^b \left(1 - \frac{1}{k(t)}\right) A(t)(q^*(s))^{-1/k(t)}g(t) dt - c_1 - c_2 \right] q^*(s) \\ &= \int_a^b \left(1 - \frac{1}{k(t)}\right) A(t)(q^*(s))^{1-1/k(t)}g(t) dt - (c_1 + c_2)q^*(s), \end{aligned}$$

which is a concave function of  $q^*(s)$ . Therefore, the  $q^*(s)$  that maximizes  $\pi_\ell(s)$  is uniquely determined by the first-order condition, from which we have (14). Substituting (14) into (12), we can have the corresponding optimal transportation fee, which is uniquely given by (13). This completes the proof.  $\square$

**Proof of Theorem 5.** Following Theorem 1 and Corollary 1, we can obtain the optimal pricing decision (18) in the second stage.

Substituting (18) into (16), we express the profit as a function of  $q_c$  as follows:

$$\begin{aligned} \Pi_c(q_c) &= \mathbb{E}_t \{ p_c^* \mathbb{E}_\varepsilon \{ \min(q_c m(t), D(p_c^*, \theta(t))) \} \} - (c_1 + c_2)q_c \\ &= \int_a^b \frac{k(t)}{k(t)-1} A(t)q_c^{1-1/k(t)}g(t) dt - (c_1 + c_2)q_c. \end{aligned}$$

Clearly,  $\Pi_c(q_c)$  is concave in  $q_c$  because  $k(t) > 1$ . Therefore, the optimal shipping quantity is uniquely determined by the first-order condition. By letting

$$\frac{d\Pi_c(q_c)}{dq_c} = \int_a^b A(t)q_c^{-1/k(t)}g(t) dt - (c_1 + c_2) = 0,$$

we know that the optimal  $q_c^*$  solves Eq. (19). This completes the proof.  $\square$

**Proof of Theorem 6.** Based on the proof of Theorem 3, we have

$$\begin{aligned} \left. \frac{d\pi_m(q)}{dq} \right|_{q=q_c^*} &= \int_a^b \left(1 - \frac{1}{k(t)}\right) A(t)q_c^{-1/k(t)}g(t) dt - (c_1 + s\gamma_0) \\ &\leq \int_a^b A(t)q_c^{-1/k(t)}g(t) dt - (c_1 + s\gamma_0) \end{aligned}$$

$$\begin{aligned} &= (c_1 + c_2) - (c_1 + s\gamma_0) \quad (\text{due to Theorem 5(ii)}) \\ &= c_2 - s\gamma_0 < 0, \end{aligned}$$

which implies that  $q_c^* > q^*$ . This completes the proof.  $\square$

**Proof of Proposition 1.** For ease of exposition, we define a function as follows:

$$\begin{aligned} \Phi(x, t|q) &= x\mathbb{E}_\varepsilon[\min(y(t)x^{-k(t)\varepsilon}, qm(t))] \\ &= xqm(t) - x \int_0^{qm(t)/y(t)x^{-k(t)}} (qm(t) - y(t)x^{-k(t)\xi})f(\xi) d\xi. \end{aligned}$$

Because both  $y(t)x^{-k(t)}$  and  $qm(t)$  are decreasing in  $t$ , it is trivial that  $\Phi(x, t|q)$  is decreasing in  $t$ . Taking derivative with respect to  $x$ , we have

$$\frac{\partial\Phi}{\partial x} = \left\{ y(t)x^{-k(t)} \int_0^z \xi f(\xi) d\xi \left[ \frac{z\bar{F}(z)}{\int_0^z \xi f(\xi) d\xi} - (k(t)-1) \right] \right\} \Big|_{z=qm(t)/y(t)x^{-k(t)}}$$

As

$$\left( \frac{z\bar{F}(z)}{\int_0^z \xi f(\xi) d\xi} \right)' = \frac{\bar{F}(z)}{(\int_0^z \xi f(\xi) d\xi)^2} \int_0^z (h(\xi) - h(z))\bar{F}(\xi) d\xi \leq 0,$$

due to Assumption 1, we know that  $\Phi(x, t|q)$  is quasi-concave in  $x$  and the optimal  $x$  is given by

$$x^* = \left( \frac{z^*(t)y(t)}{qm(t)} \right)^{1/k(t)},$$

where  $z^*(t)$  satisfies (6) (recall Theorem 1). Let  $\Psi(t|q) = \Phi(x^*, t|q)$ , we then have

$$\frac{d\Psi(t|q)}{dt} = \frac{\partial\Phi(x, t|q)}{\partial x} \Big|_{x=x^*} \frac{dx^*}{dt} + \frac{\partial\Phi(x, t|q)}{\partial t} = \frac{\partial\Phi(x, t|q)}{\partial t} \leq 0,$$

which implies that  $\Psi(t|q)$  is decreasing in  $t$ . Consequently, by conducting some algebra transformations, we have

$$s(q, t) = c_2 + \beta(1-\alpha) \left[ \frac{1}{q} \Psi(t|q) - (c_1 + c_2) \right],$$

which is decreasing in  $t$ . This completes the proof.  $\square$

**Proof of Theorem 8.** We first investigate the distributor’s optimal decisions. Only the retail price needs to be determined, because according to the WMC contract, the distributor should purchase all of the producer’s marketable quantity at wholesale price. The distributor’s expected profit is

$$\pi'_d(p|q, t) = \mathbb{E}_\varepsilon \{ p \min(qm(t), y(t)p^{-k(t)\varepsilon}) - w(q, t)qm(t) \}.$$

Again, by defining a stocking factor  $z := qm(t)/y(t)p^{-k(t)}$ , we transform the decision variable  $p$  into  $z$ , and the profit function becomes

$$\pi'_d(z|q, t) = \left( \frac{zy(t)}{qm(t)} \right)^{1/k(t)} \mathbb{E}_\varepsilon \left\{ \min\left(1, \frac{\varepsilon}{z}\right) \right\} qm(t) - w(q, t)qm(t).$$

Clearly, the optimal stocking factor that maximizes  $\pi'_d(z|q, t)$  should be  $z^*(t)$  (refer to Theorem 1), and the distributor’s optimal profit becomes

$$\begin{aligned} \pi'_d(z^*(t)|q, t) &= \alpha \left[ \left( \frac{z^*(t)y(t)}{qm(t)} \right)^{1/k(t)} \mathbb{E}_\varepsilon \left\{ \min\left(1, \frac{\varepsilon}{z^*(t)}\right) \right\} qm(t) - (c_1 + c_2)q \right] \\ &= \alpha \left[ \frac{k(t)}{k(t)-1} (z^*(t)y(t))^{1/k(t)} m(t)^{1-1/k(t)} \bar{F}(z^*(t)) q^{1-1/k(t)} - (c_1 + c_2)q \right] \\ &= \alpha \left( \frac{k(t)}{k(t)-1} A(t)q^{1-1/k(t)} - (c_1 + c_2)q \right), \end{aligned}$$



where the first equality is due to (20), the second equality is due to (6), and the last equality is due to  $A(t) = (z^*(t)y(t))^{1/k(t)} m(t)^{1-1/k(t)} \bar{F}(z^*(t))$ .

We next investigate the producer's optimal decisions. Note that the wholesale price is not a decision variable because it is already determined by the WMC contract. Therefore, the producer only needs to determine the shipping quantity  $q$ . The producer's expected profit is

$$\begin{aligned}\pi'_m(q) &= \mathbb{E}_t\{w(q,t)qm(t) - [c_1 + s(q,t)]q\} \\ &= (1-\beta)\mathbb{E}_t\{w(q,t)qm(t) - (c_1 + c_2)q\} \\ &= (1-\beta)(1-\alpha)\mathbb{E}_t\left\{\frac{k(t)}{k(t)-1}A(t)q^{1-1/k(t)} - (c_1 + c_2)q\right\} \\ &= (1-\beta)(1-\alpha)\Pi_c(q).\end{aligned}$$

Therefore, the shipping quantity that maximizes  $\pi'_m(q)$  is equal to the optimal shipping quantity of the centralized supply chain,  $q_c^*$  (refer to Theorem 5).

We have shown that under the proposed contracts, (i) the producer's shipping quantity is  $q_c^*$ ; (ii) there is no product outflow (to the outside of the system under consideration); and (iii) the distributor's optimal retail price is also the same as that of the centralized supply chain for any  $t$  (because the optimal stocking factor remains unchanged). Therefore, the decentralized supply chain acts the same as the fully centralized supply chain. This completes the proof.  $\square$

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