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A Multi-attribute Reverse Auction Decision Making Model Based on Linear Programming

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Abstract

Because of the popularity of the Internet, e-commerce is used more and more widely around the world. At the same time, the business style of procurement has also changed a lot. Under this circumstance, online reverse auction came into being. Based on the theoretical knowledge and practical status quo, this paper has studied some key issues, such as the applicable conditions and the basic process of multi-attribute reverse auction, especially focused on the mechanism of decision-making in the process of performing multi-attribute reverse auction. Again in this paper, AHP method has been employed to determine the weight of each attribute; moreover, based on the linear programming theory, a multi-attribute reverse auction model has been established. Finally, a calculation example has been conducted to demonstrate the utility and availability of the model.

Keywods: reverse auction; decision making; AHP; linear programming

1. Introduction

At the end of 2010, Geoffrey Boyd, who comes from the reverse auction website Priceline, has won the 2010 year's "wealth creator" by the Journal of *Chief Executive Officer*, which implies that the reverse auction contains infinite business opportunity. According to a survey launched by e-Bay, a successful reverse auction can reduce the cost level of procurement between 6.3% and 43%, averagely 18%. However, the practice of reverse auction is far more on top of its theories, which are quite conceptive and scattered. Jap (2002) argued that the motive for a business to take reverse auction comes from saving money, enhancing efficiency and application of emerging technology [1]. From the perspective of supply chain, Wu and Li (2007) scrutinized why and how the reverse auction can reduce total cost [2]. Moreover, Stephan and Andreas (2004) proposed that the exact specification of product, sufficient preparation of buyer as well intense competition between suppliers are the key elements to determine the success of reverse auction[3]. Additionally, Bichler (2002) defined multi-attribute auction as one kind of auction patterns in

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which more attributes other than price should be considered [4]. Dekrajanpetch (2000) firstly attempted to employ the linear programming method to solve multi-attribute on-line auction problems [5]. Enlightened by the previous theoretical tools and based on the demand of current practice of reverse auction, this paper established a quantified multi-attribute model to disclose the inherent mechanism of decision making in the process of reverse auction, which is intended to facilitate the stakeholders to optimize and expedite their decision making as well take appropriate actions.

2. Basic theory of multi-attribute reverse auction

2.1. The connotation of multi-attribute reverse auction

The so-called reverse auction is opposite to daily said auction, namely, it is an on-line bidding mechanism, in which a buyer and many sellers are involved. Besides price, many other attributes in the purchase (including the quality, delivery date, supplier prestige, etc) are taken into consideration. Moreover, many factors could affect the establishment of multi-attribute reverse auction mechanism, such as : 1) the contents and types of goods or services; 2) ways of trading, including B2B, B2C, GC or C2C; 3) buyer's preference; 4) approaches of tender; 5) degree of matching between buyer and sellers; 6) feedback of both buyer and sellers.

Compared to traditional personnel purchase mode, reverse auctions have great advantages, such as: 1) seeking tenders and bid via Internet can increase transparency of the trading, save communications costs and time greatly and offer more choices for buyer; 2) reducing goods' price and enhance its quality effectively and cutting costs of transaction costs to a lowest level; 3) facilitating to eliminate regional differences. As to the multi-attribute reverse auctions, it can overcome the drawbacks of previous pattern of transaction, in which price is the only factor to be concerned about, and lead to an optimal match between demand and supply.

Nevertheless, multi-attribute reverse auctions are not appropriate for all purchase activities, especially for those under the circumstances that the internet system is not well developed and credibility system is not cultivated sufficiently.

2.2. Specific process of multi-attribute reverse auction

In a reverse auction activity, a buyer should make a thorough preparation in advance to make a reasonable decision, and perform controlling and monitoring in the process timely to attain an acceptable result. After the auction activity is finished, the buyer should summarize related experiences to make a better decision next time. Generally speaking, the steps of a typical preparation for the auction mainly include: 1) determining products' attributes and describing them exactly; 2) selecting appropriate professional website referred to e-commerce; 3) releasing demand information to the website timely and properly. In a bidding activity, the buyer should monitor the whole process timely, once he or she finds a malicious bidding (a cheating bid) or discrepancy, he or she should take remedial measures immediately to rule out the bidders or terminate the bid. The holistic process of multi-attribute reverse auction is illustrated in Fig. 1.

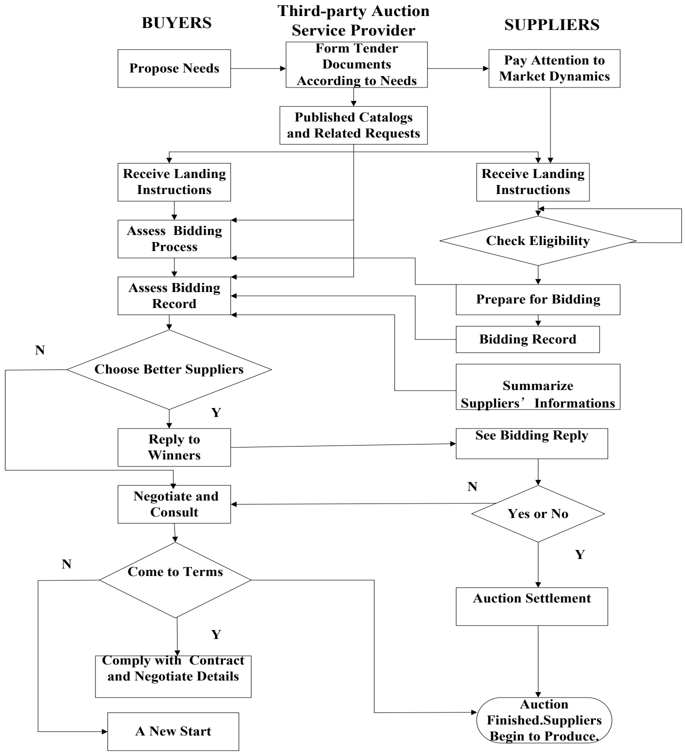

Fig. 1 Processes of multi-attribute reverse auction

3. Decision-making model of multi-attribute reverse auctions

Considering the buyers' personal preferences and specific purposes, such as bulk buying, maintaining customer relationships, or because of urgency of the task, they may choose more than one (non exclusive) winners from all candidate suppliers. Therefore, we attempt to adopt linear programming theory to set up a multi-winners' decision-making model. The conceptual framework of the model could be described as: according to their actual needs, buyers offer instructions of goods' attributes, and then could figure out the weight of each attribute respectively; as well they also need to establish the utility function at each level for individual attribute (scoring based on the function); finally, the candidate suppliers with high scores will be chosen to be winners in the reverse auction.

3.1. Assumptions

In order to simplify the research work, we assume that: 1) a buyer needs to purchase a kind of product and will choose a couple of suppliers, namely, the number of winners is more than one. 2) in practice, the bidders might consider the effect of economy of scale and other factors, so the number of tender is inseparable. 3) each attribute can be quantified. Actually, there are some indicators are necessary but hardly to be quantified, such as rating of supplier's credit and so on. Usually, fuzzy appraisal or expert scoring method could be employed to deal with this problem. Whatever is adopted, enough qualified experts should be selected in advance. 4) the candidate suppliers cannot be aware of the buyers' base price and utility functions referred to relevant attributes. 5) there is no expenditure on delivery or transportation service for buyers.

3.2. The model

The first step is to determine the relative importance of key attributes, namely the weights, by AHP method.

1) Establishing the main hierarchies. Taking the frequently used factors (price, quality, supplier credit, delivery date, payment methods) as main attributes, we can construct a hierarchy chart shown in Fig.2.

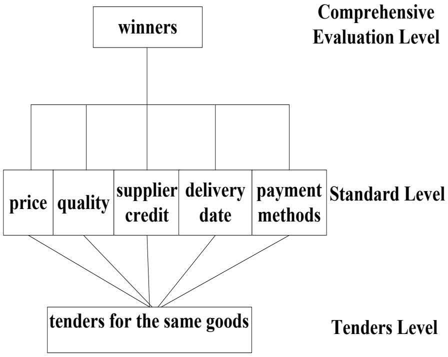

Fig.2 Hierarchy chart

2) Constructing the attribute judgment matrixes. A common ratio scale (shown in Table 1) is employed.

Table 1. Ratio scale values

NO.	Rating of Importance	Weight Assignment
1	elements *i* and *j* are equally important	1
2	i is a little important than j	3
3	i is important than j	5
4	i is more important than j	7
5	i is much more important than j	9
2,4,6,8, respectively, in the middle between two adjacent value. The value of opposite meaning is reciprocal of relative value		

Assume that there are n attributes, and attribute judgment matrix of each two attributes comparison $U = (U_{ij})_{n \times n}$ could be obtained by qualified experts.

$$U = \begin{bmatrix} \mu_{11}, \mu_{12}, \dots \mu_{1n} \\ \mu_{21}, \mu_{22}, \dots \mu_{2n} \\ \dots\dots\dots\dots\dots\dots \\ \mu_{n1}, \mu_{n2}, \dots \mu_{nn} \end{bmatrix} \tag{1}$$

3) Determining the weight of each attribute.

$$M_i = \prod_{j=1}^{n} \mu_{ij}, i = 1,2,\cdots n \tag{2}$$

$$W_i = \sqrt[n]{M_i} \tag{3}$$

$$W_i' = W_i / \sum_{j=1}^{n} W_j \tag{4}$$

$W = [W_1', W_2', \dots W_n']^T$ is the target weight of each attribute.

4) Judging the compatibility of each weight. As each judgment matrix is given by each expert, there might be some discrepancies, thus it is necessary to test the consistency:

$$CI = \lambda_{\max} - n \Big/ n-1 \tag{5}$$

$$\lambda_{\max} = \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}\mu_{ij}W_j'}{W_i'} \tag{6}$$

$$CR = \frac{CI}{RI} \tag{7}$$

Where , the random consistency index RI 's are given in Table 2.

Table2. Random consistency index

Matrix's Order	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.89	1.12	1.26	1.36	1.41	1.46	1.49

Only if CR<0.1, the consistency of the judgment matrix can be testified; otherwise, appropriate adjustments should be made.

The second step is to build up an optimization model based on linear programming, shown as equation 8 and expression group 9.

$$Max.Z = \sum_j W_j'(\sum_k V_{jk}a_{ljk}) + W_p'V_p(P) \tag{8}$$

$$\begin{cases} D_{\min} \le \sum_l Q_l\rho_l \le D_{\max} & (a) \\ \rho_l \in \{0,1\} & (b) \\ R_{\min} \le \sum_l \rho_l \le R_{\max} & (c) \\ \sum_l Q_l\rho_l P_l \le L & (d) \\ S_{\min} \le \sum_k d_{jk} \le S_{\max} & (e) \\ V_{\mathrm{jk}} = \alpha + \beta X_{jk} & (f) \\ st.\sum_k a_{ljk} = 1 & (g) \\ a_{ljk} \in \{0,1\} & (h) \end{cases} \quad \text{c} \tag{9}$$

In this optimization model, Z stands for the general utility (general score), the candidate supplier(s) with highest scores will be chosen to make the deal. W_j' represents weight of attribute j ; V_{jk} represents utility of attribute j in level k ; a_{ljk} is a variable of 0-1(if supplier l can meet the attribute j at level k , then $a_{ljk} = 1$, otherwise it is 0); $D_{\min}$ and $D_{\max}$ means buyer's lowest and highest limit of demand respectively; Q_l represents tender l 's quantity supplied; ρ_l is a 0-1 variable, if candidate l win the bid, then $\rho_l = 1$, otherwise it's 0 ; $D_{\max}$ and $D_{\min}$ are the upper and lower limit of number of winners; P_l represents the price of supplier l 's tender; L is a retention value of buyers; d_{jk} is also a 0-1 variable, it represents the winners' k level of j attribute, when k appears for the first time, it is equal to 1,otherwise,it is 0; $S_{\min}$ and $S_{\max}$ are the lower and upper bound of number of levels in attribute j; X_{jk} represents the quantitative values of each level, α and β are parameters.

Usually, price greatly influence the success of purchase behavior, thus in order to highlight its outstanding position, the price is written alone in the model; expression 9(a) is a constraint to the quantity demanded; 9(b) is used to indicate the result of bidding, namely, if supplier l is chosen, it is 1, otherwise, it is 0; 9(c) is a constraint to the number of winners, in the process of reverse auction, fewer suppliers means great risk, but a large number of suppliers will cause great problems of coordination; 9(d) is a constraint to the retention value L, because it is necessary for the buyers to constrain their budget of procurement; 9(e) is a constraint to products' homogeneity, which is to limit the level of some attributes; 9(f) means that each level in each attribute has its linear utility function (score value); 9(g) is to guarantee that each supplier only provides one level for each attribute in a tender; 9(h) means: if supplier L offers level k in attribute j, then the variable is equal to 1.

3.3. Examples

Company A is a listed company in Wuhan city, China. In order to satisfy the needs of R&D activities, needs to purchase a batch of laptop computers. The lowest limit of total amount is 1000 units, while the highest limit is 1200units. The amount that individual bidder can offer varies from 300 to 600. The specific requirements for the laptops are shown in Table 3. Here, the weight of each attribute has been determined in advance, and each attribute can be assessed by a linear scoring function.

Table 3.Specific requirements

attributes	requirements	weights	scoring functions	function values
color	black; blue; red	0.05	black:S_{11}=100 blue:S_{12}=80 red:S_{13} =60	S_{11}=100;S_{12}=80;S_{13}=60
Hard Disk(G)	80	0.2	S_2=100	100
Dominant Frequency(G)	2.0; 2.2; 2.4	0.1	S_3=100x-140	60;80;100
supply time (Day)	10～30(integer)	0.15	S_4=120-2t	Depending on the value of t
price(RMB10K)	0.6 ≤P≤1	0.5	S_5=220-200p	Depending on the value of p

After filtration, there are five suppliers come from 35 bidders can preliminarily satisfy the buyers' demand. The details of their biddings are shown in Table 4.

Table 4 .The details of five biddings

NO.	S_1	S_2	S_3	S_4	S_5	quantity
1	black	80G	2.0	18	0.8	500
2	red	80G	2.4	22	0.88	450
3	blue	80G	2.2	20	0.85	300
4	red	80G	2.4	25	0.86	400
5	black	80G	2.2	18	0.9	300

According to the principle of our linear programming model, we could establish a specific multi-attribute reverse auction model as shown in formula (10) and (11). The candidate with larger Z value will be taken into consideration preferentially.

$$Max.Z = 0.05\sum_{k_1} V_{1k_1}a_{1k_1} + 0.2\sum_{k_2} V_{2k_2}a_{2k_2} + 0.1\sum_{k_3} V_{3K_3}a_{3k_3} + 0.15\sum_{k_4} V_{4k_4}a_{4k_4} + 0.5\sum_{k_5} V_{5k_5}a_{5k_5} \quad (10)$$

$$\begin{cases} st.\sum_k a_{ljk} = 1 \\ 2 \le \sum_l \rho_l \le 3 \\ 300 \le Q_l \rho_l \le 600 \\ 1000 \le \sum_l Q_l \rho_l \le 1200 \\ a_{ljk} \in \{0,1\} \\ \rho_l \in \{0,1\} \\ V's\ functions\ \ have\ been\ given \end{cases} \qquad \text{C} \qquad (11)$$

Through calculation, we can obtain the orders of the given 5 bidders according to their respective Z score: $Z_1 > Z_3 > Z_4 > Z_2 > Z_5$, so the buyer firstly assign 500 units to bidder 1, then 300 units to bidder 3 and 400units to bidder 4, while the total amount is just equal to 1200 units, which attains the upper bound of buyer's quantity demanded. Therefore, bidder 1, 3, 4 are winners, while bidder 2 and bidder 5 have to be eliminated.

4. Conclusions

In this paper, we combine AHP approach with linear programming method to propose a multi-attribute reverse auction model, which is intended to be used to raise the efficiency and accuracy of decision making in the action of reverse auction. Compared to previews research, more constraints have been considered and the determination of weight of each attribute might be more scientific and reasonable. Actually, assisted with more professional software package, this model could be more complete and close to the real situations.

Our model is based on a very strict and ideal assumptions, i.e., a buyer purchase only one product or service, there is no transportation cost for buyers, etc. Actually, the actions in a real reverse auction is much more complex than the theoretical analysis, namely, a buy is quite probably purchase more than one goods or service on time, and the expenditure could not be neglected. Moreover, the weight of each attribute is really very hard to be determined everywhere and at anytime without perfect knowledge and enough experts. Nevertheless, it's the first attempt to conduct a quantitative research on multi-attribute reverse auction problem, and the basic methodology of decision making could be helpful to solve this problem, and in the future we'll extend our work to a more realistic and sophisticated occasion. Thus, we believe our research is interesting and meaningful.

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