Accepted Manuscript

An analytical model of the relationship between product quality and advertising

Régis Chenavaz, Sajjad Jasimuddin

PII: S0377-2217(17)30444-7
DOI: 10.1016/j.ejor.2017.05.016
Reference: EOR 14444

To appear in: *European Journal of Operational Research*

Received date: 8 June 2016
Revised date: 8 May 2017
Accepted date: 10 May 2017

Please cite this article as: Régis Chenavaz, Sajjad Jasimuddin, An analytical model of the relationship between product quality and advertising, *European Journal of Operational Research* (2017), doi: 10.1016/j.ejor.2017.05.016

Highlights

- Optimal control model with advertising and product quality.
- Explanation of both positive and negative advertising-quality relationships.
- Proof of the conjecture of Tellis and Fornell (1988) as special case.
- We provide testable empirical implications.

An analytical model of the relationship between product quality and advertising

Régis Chenavaz*
Kedge Business School, AMSE, CNRS & EHESS
Domaine de Luminy, Rue Antoine Bourdelle
13288 Marseille, France
regis.chenavaz@kedgebs.com

Sajjad Jasimuddin
Kedge Business School
Domaine de Luminy, Rue Antoine Bourdelle
13288 Marseille, France
sajjad.jasimuddin@kedgebs.com

Abstract

The existing literature debates if the products of better quality are more heavily advertised. This article resolves this contradiction by answering the question of when better quality leads to more advertising. It provides a novel articulation of prior empirical research, modeling the advertising-quality relationship in an optimal control setting. On the supply-side, a firm carries out advertising to promote its product and product innovation policies that improves product quality. On the demand-side, consumers are sensitive to product price, product quality, and advertising expenditure. The paper identifies the conditions that will dictate when the advertising-quality relationship will be positive or negative. The argument is that advertising increases with quality (i.e., positive relationships) if the demand effects (quality and advertising effects on demand) outweigh the supply effect (quality effect on cost). Alternatively, advertising decreases with quality (i.e., negative relationships) if the demand effects are lower than the supply effect. Consequently, despite consumer awareness of quality, a firm may advertise a product of lower quality more to maximize profit.

Keywords: Dynamic advertising, product quality, advertising-quality relationship, marketing-mix, optimal control

*Corresponding author. Tel: +33 (0)491 827 318, Fax: +33 (0)491 827 983.

1 Introduction

A stream of literature discusses the notions surrounding quality and advertising of a product isolatedly. Since the term "marketing-mix" was first coined by Neil Borden, the president of the American Marketing Association in 1953, several scholars (Banting and Ross, 1973; Kotler and Keller, 2006) have popularized the term incorporating four Ps (i.e., price, product, promotion, and place). The product quality through innovation and its promotion through advertising often plays a crucial role for corporate success. The fact that the information about product quality is spread by advertising and word of mouth (Kalish, 1985). Scholars (e.g., Crosby Philip 1984; Deming 1982; Kalish 1985) discuss the linkages among key strategic variables such as quality, markets share, price, profitability, advertising expenditures and customer service. For example, Kalish (1985) introduces a framework for modeling innovation diffusion that includes price and advertising. In judging product quality, consumers use information about quality-related product features through advertising, price or brand name (Jerry, 1977).

There is a vast literature that analyzes the advertising-quality relationship. Most specifically, authors (e.g., Orzach et al. 2002; Kirmani and Wright 1989) deal with the influence of advertising on the product's quality. Consumers simply perceive a correlation between advertising expense and quality in some markets (Kirmani and Wright, 1989). Kirmani and Wright (1989) explore that perceived advertising expense affects quality expectations. Orzach et al. (2002) examine the role of advertising expenditures as signals of quality. In this regard, by predicting the existence of a group of heavy advertisers and of a low advertiser, Chioveanu (2008) suggests that high advertisers tend to have higher prices and argues that consumers perceive highly advertised brands as different. Similarly, Piga (2000) develops a model in which he contends that it is necessary to consider the joint action of product quality and advertising, in which firms can step up their quality but, at the same time, limit a fall in price by simply investing in advertising.

Although the extant literature focuses on advertising as signals of quality, this paper explores the impact of product quality on advertising expense. It is to be noted that quality and advertising have typically been incorporated into the existing frameworks in a rather ad hoc way. They do not, however, explicitly focus on the impact of a product of better quality on such marketing decision variable as advertising expenditures. Rather the existing research only debates if products of better quality are more heavily advertised. Since product quality is an important component of competitive strategy (Narasimhan and Ghosh, 1994), this paper intends to look the advertising from a different angle by addressing the influence of product quality on advertising. In other words, the article attempts to answer the question of when (as opposed to whether) a product of better quality is more advertised than a product of lower quality.

Most specifically, this paper develops a model of the advertising-quality relationship in an optimal control setting by building on Tellis and Fornell (1988). The objective of this study is to suggest a model that formally derives both positive and negative advertising-quality relationships from demand- and supply-sides effects. The reminder of the paper is organized as follows. Section 2 illustrates the relationship between advertising and product quality along with the underlying assumptions in the selected studies. The next section highlights the formulation of the model. Section 4 discusses the model. The paper concludes with a general contribution in Section 5.

2 Related Literature and Contributions

The fact that a large part of research identifies the role of advertisement on the product quality. It is to be noted that the analysis, based on optimal control, links supply and demand to firm organization and consumers preferences (Feichtinger, 1982; Feichtinger et al., 1994; Jørgensen et al., 2006, 2009). Similarly, new product diffusion has been extensively studied by several researchers (e.g., Dawid and Feichtinger 1995; Feichtinger 1992; Krishnan and Jain 2006; Sethi et al. 2008; Krishnamoorthy et al. 2010; Swami and Dutta 2010; Fruchter and Van den Bulte 2011; Chutani and Sethi 2012; Helmes et al. 2013; Yenipazarli 2015; El Ouardighi et al. 2016a,b). Production cost is based on product quality. A monopoly simultaneously conducts dynamic advertising and product innovation policies. Product innovation helps to improve product quality (Vörös, 2006; Li and Ni, 2016; Pan and Li, 2016; Chenavaz, 2016). It can be argued that a product of better quality is more advertised than a product of lower quality. Similarly, demand of consumers augments with advertising expense and product quality. This work thus builds on literature relating to advertising-quality relationship and dynamic advertising.

It is found that a large part of the theoretical literature considers either informative or persuasive advertising. Taking the informative perspective into account, the advertising-quality relationship is positive. In this regard, Nelson (1974) identifies three effects associated with the informative view which include signaling-efficiency, repeat-business, and match-products-to-buyers effects. According to the signaling-efficiency effect, an efficient firm, characterized by lower production cost, has greater incentives to create demand by providing better quality and more advertising (Kihlstrom and Riordan, 1984; Kirmani, 1997; Hertzendorf and Overgaard, 2001; Fluet and Garella, 2002; Linnemer, 2002, 2012; Horstmann and Moorthy, 2003). Following the repeat-business effect, a firm advertises a better quality product more as this product generates additional future purchases (Schmalensee, 1972, 1978; Milgrom and Roberts, 1986; Hertzendorf, 1993; Horstmann and MacDonald, 1994; Moraga-González, 2000; Zhao, 2000; Orzach et al., 2002). The match-products-to-buyers effect states that a better quality product is more heavily advertised so that it matches consumers who most value its quality (Grossman and Shapiro, 1984; Bagwell and Ramey, 1993; Meurer and Stahl II, 1994;

Johnson and Myatt, 2006; Anderson and Renault, 2006). It thus becomes important to make an attempt to formulate NelsonŠs basic ideas in a complete, formal model incorporating both the quality and advertising decisions. In fact, a number of authors since Nelson have investigated the relationship between quality and the use of the non-informative or image advertising on which he focused, and some have been explicitly interested in formalizing his ideas. Conversely, in the persuasive view, the advertising-quality relationship is negative. Indeed, Dorfman and Steiner's condition, as cited in Comanor and Wilson (1979), suggests a negative relationship as high advertising may be used to increase consumer preferences for low quality goods. If the quality of product is endogenous, lower quality firms may use more efficient advertising technologies so as to attract more potential customers (Colombo and Lambertini, 2003). A major part of this literature considers advertising implications over time with parametric models, as assessed in the surveys by Huang et al. (2012) and Jørgensen and Zaccour (2014). Following a parametric approach, Doganoglu and Klapper (2006) emphasize the importance of advertising intensity. More recently, Chioveanu (2008) shows that persuasive advertising softens competition and drives higher price dispersion.

Extensive empirical research has, therefore, been conducted to arbitrate between both viewpoints (Thomas et al., 1998; Moorthy and Zhao, 2000; Ackerberg, 2001, 2003; Tsui, 2012). However, the empirical findings of the existing research imply little or no systematic relationship between advertising and quality. Such mixed support reflects the contingency of the advertising-quality relationship that is linked to demand and supply circumstances.

There is another stream of research that focuses on dynamic advertising, which was first investigated by scholars such as Nerlove and Arrow (1962). In this context, Piga (1998, 2000) explains dynamic advertising together with product differentiation and sticky prices. Several other authors (e.g., Erickson 2009; Grosset et al. 2011) analyze the goodwill impact, while Jørgensen et al. (2009) looks at the effect of an entertainment event on the advertising policy. The competition between national and store brands that affects advertising is analyzed by Karray and Martín-Herrán (2009). Gupta and Di Benedetto (2007) consider the threat of competitive entry.

The extant literature reports a controversial relationship between quality and advertising decisions. Some studies support the theory that higher quality has positive linkage/ result in higher advertising expenditures because advertising is an indicator of the quality of the advertised products (Nelson, 1974; Bagwell and Ramey, 1994). However, other empirical studies showed that the relationship between quality and advertising is negative.

Table 1: The Relationship between Advertising and Product Quality along with the Underlying Assumptions in Selected Studies

Authors	Research	Link	Product	Assumptions	Results
Abe (1995)	Theoretical	+	Any product of national brand and private label clone	A consumer is determined to make a purchase of either a national brand or its private label clone. He/she has a prior belief that the national brand is high quality type. He/she assigns different utility values to the two types of the national brand.	A consumer can purchase of either a national brand or its private label clone. The manufacturer of high quality product advertises more than it would if its product were similar to the clone in quality.
Ackerberg (2003)	Empirical	+	Nondurable, experience good (grocery product)	Advertising intensity and use experience signal product quality.	The results suggest that in this market, advertising's primary effect is that of informing consumers.
Archibald et al. (1983)		+	The runner's shoe	Published third party ratings on product quality is used as measures of high and low quality products.	Positive linkage between quality and advertising with third party ratings which shift experience goods towards search goods.

Caves (1986)	Empirical	+	The branded goods	High quality generally induces higher advertising expenditures.	The determinants of information outlays (media advertising, sales force, other sales promotion) include buyers' overall demands for information, buyers' access to sources not controlled by the seller, the relative efficiency of seller-supplied information, and competitive conditions in the product market.
Caves and Greene (1996)	Empirical	+/-	Multiple product	Quality-price correlations increase with the scope for vertical differentiation and decline for innovative and convenience goods. Positive quality-advertising associations mainly reflect verifiable information about quality attributes.	There is a positive correlation between advertising and quality when examining goods where buyers' experience and search are effective at guiding brand choice but a negative correlation for convenience goods. The empirical results fail to find a systematic and significant positive correlation between advertising and quality on average in a cross-section of industries.
Comanor and Wilson (1979)	Empirical	-	Multiple product	Advertising accentuate preferences for specific brands even though physical features are not different. Advertising differentiates products.	Low quality products set higher advertising levels to counter their relative product disadvantages.

Erdem et al. (2008)	Empirical	+	Ketchup	The relationships among price, advertising frequency, and quality are specified that are assumed hold in equilibrium. Households are assumed to know these equilibrium relationships and to use them to help infer brand quality.	Price is an important quality-signaling mechanism. The role of advertising frequency in signaling quality is also significant, but it is less qualitatively important than price.
Kash and Miller (2009)	Empirical	-	Nursing home	Nursing homes should engage in advertising to improve awareness of the services offered in a particular market and to signal high-quality services.	Advertising expenses are not associated with better nursing home quality. More advertising expenditures are not necessarily associated with better quality, consumers can be misled by advertisements and choose poor quality nursing homes.
Kihlstrom and Riordan (1984)	Theoretical	+		Advertising signals quality in the short term. Long run reputation is an asset to a high quality firm. Return to advertising is greater for higher quality product because of repeat purchases.	A continuum of equilibria exists. Quality is positively related to advertising.

Milgrom and Roberts (1986)	Theoretical	+		Both advertising and price signal quality.	The unique separating equilibrium exists where the low-quality firm picks its full-information optimum and the high-quality firm sends just enough signaling (more advertising) to distinguish itself.
Moorthy and Hawkins (2005)	Empirical	-	Household items (e.g., cookware, overcoat, nasal spray, yogurt)	Consumers can infer that high-quality products would advertise more than low-quality products.	Higher advertising spending signals lower quality because the firm is desperate to spend so much on advertising.
Nelson (1975)	Empirical	+	Multiple product	Markups and marginal costs are the same for low and high quality products. Return to advertising is greater for higher quality.	For experience goods, consumers should rationally infer that only high-quality products would spend much in advertising. High quality products tend to advertise more. Quality and advertising are positively related.

Nichols (1998)	Empirical	+	Automobiles	Advertising serves as a signal of higher quality.	The results reveal that advertising has a role to provide information and signal quality to imperfectly informed consumer. Advertising-quality linkage is found by examining how advertising levels vary with a quality measure that is not observable at the time of purchase.
Orzach et al. (2002)	Theoretical	-	Household items	Modest advertising signals strength and high quality.	A high quality firm may choose to signal its identity by lowering introductory advertising expenditures to a level below that of the low quality firm. Consumers respond favorably to advertising cuts and correctly identify quality.
Schmalensee (1978)	Theoretical	+/-		Cost and price are equal for all the firms. All sellers are assumed to charge the same price. All firms face the same cost function.	The relationship between advertising and quality depends on the cost advantage of producing low quality goods and the effectiveness of advertising. Low quality producers may use advertising because markups are negatively correlated with quality.

Song et al. (2015)	Empirical	+/-	Movies	The high-quality firm cannot effectively signal its product quality through the amount of its advertising during the prelaunch period.	The relationship between advertising and product quality depends upon pre-launch and post periods. The results show that post-launch advertising is a reliable quality indicator and increases revenues, whereas pre-launch advertising is not a reliable quality indicator, even if it leads to higher revenues.
Tellis and Fornell (1988)	Empirical	+/-	Multiple product	Two factors influence the correlation between advertising and quality level: (i) amount of product information consumers have and (ii) cost of producing high quality goods.	The relationship between quality and advertising depends upon product life cycle. For early product life cycle, the relationship is positive. For mature product life cycle, the relationship is negative.
Thomas et al. (1998)	Empirical	+	Automobiles	Manufacturers use both price and advertising to signal the quality of their products.	The firm of a high quality product also uses advertising to signal product quality. The high quality product has higher advertising expenditures than lower quality ones.

Table 1 depicts that some positive and significant correlations among quality and advertising exist in a cross-section of products or models of a product within an industry. In many instances, there seems to be no correlation at all among the two variables or even negative correlation. As mentioned earlier, the majority of the previous literature considers the advertising-quality relationship as a one-dimensional issue. Interestingly, Tellis and Fornell (1988) explain this relationship as a two-dimensional matter. They conjecture that the advertising-quality relationship may be negative if (1) product quality is produced at high cost and (2) consumers respond strongly to advertising. Such a negative relationship is explained by assuming unknown product quality and heterogeneous production cost of competing firms. Tellis and Fornell (1988, p. 66) contend that consumers may be misled about quality and firms are heterogeneous, stating: "Because of the uncertainty about quality, consumers will be responsive to advertising (...). If quality costs substantially more to produce, the low quality producers, with lower costs, will advertise heavily enough to attract a larger share of consumers (...). Consequently, lower quality would lead to higher levels of advertising."

Tellis and Fornell (1988) conjecture that advertising increases with the quality of product, provided quality is improved at a lower cost and consumers respond cautiously to advertising. Although these authors acknowledge that consumers may be misled by unknown quality, they provide no formal guarantee of their results. Our paper provides a rule for the advertising-quality relationship. The model at the base of the rule builds on the properties of the general functions of demand, cost, and innovation, yielding structural results as opposed to parametric results. The underlying model is thus loosely constrained. It accounts for both informative and persuasive views of advertising. The rule of advertising-quality relationship identifies four effects, three on the demand-side (the direct advertising and quality effects, and the indirect advertising effect) and one on the supply-side (the quality effect on production cost). That is, the paper formally considers the impact of quality on advertising as a four-dimensional problem, which seems to provide better understanding of the controversy on this impact.

This paper provides better understanding of the controversy on the relationships between quality of a product and advertising, suggesting the conditions under which this linkage will be positive or negative. The paper provides a novel articulation of prior empirical research by proposing the sign of this connection being explained by the relative weight of the demand and supply effects. This article sets up a continuous-time model of a monopolist who faces a world with quality that evolves dynamically as a function of innovation investment (presumably R&D or service) and advertising expenditure. Consumer demand in every instant, in turn, depends on advertising and quality at that instant. The positioning of the paper is to derive testable implications, so the proposed contribution is in putting some structure on the "it depends" result. This article represents the firrst theoretical foundation to both positive and negative relationships between advertising and quality.

3 Model Formulation

It thus becomes important to make an attempt to formulate Nelson's basic ideas in a complete, formal model incorporating both the quality and advertising decisions. In fact, a number of authors since Nelson have investigated the relationship between quality and the use of the non-informative or image advertising on which he focused, and some have been explicitly interested in formalizing his ideas. This paper develops a model, suggesting that it is necessary to consider the joint action of quality and advertising.

3.1 Notations

Table 2 defines the notations used in the model analysis. A dot above a variable states for the time derivative; a subscript under a variable notes the derivative with respect to that variable.

Table 2: Notation

T	= planning horizon
t	= time
r	= interest rate
$a(t)$	= advertising expense at time t (decision variable)
$u(t)$	= product innovation at time t (decision variable)
$q(t)$	= product quality at time t (state variable)
$\dot{q}(t)$	$= dq(t)/dt = K(u,q) =$ quality dynamics at time t
$\lambda(t)$	= current-value adjoint variable at time t
$C(q)$	= unit production cost
$D(a,q)$	= demand
$P(D)$	= unit price
$\pi(t)$	= current profit at time t
$H(a,u,q,\lambda)$	= current-value Hamiltonian

3.2 Model Development

Monopoly behavior is modeled in an optimal control setting. The planning horizon $T > 0$ is finite and fixed. The time $t \in [0,T]$ is continuous.

3.2.1 Quality Dynamics

The seminal work of Levy (1965) and its extension by Dutton and Thomas (1984) show that quality is tied to both induced learning and autonomous learning. Induced learning results from conscious managerial actions, such as current investment in innovation, whereas autonomous learning originates from a "practice makes perfect" effect, which may be linked to the cumulative quality (See the survey of Li and Rajagopalan 1998).

First theoretical modeling of induced learning is provided by Arrow (1962) and Dorroh et al. (1994). Empirical evidence of induced learning is provided by Sheshinski (1967) and Deming (1982). Early theoretization of autononomous learning include Womer (1979) and Spence (1981). Empirical evidence of autonomous learning is first made in the aircraft industry as recalled by Li and Rajagopalan (1998). Since then, induced learning has been repeatedly observed in numerous industries as attested by the surveys of Yelle (1979) and Fine (1986).

In this model, the firm chooses the level of innovation (or product innovation) $u(t) \in \mathbb{R}^+$ that improves quality (or product quality) $q(t) \in \mathbb{R}^+$. Thus, innovation $u(t)$ is a decision (or control) variable and quality $q(t)$ is a state variable. The quality dynamics evolve according to

$$\dot{q}(t) = K(u(t), q(t)), \tag{1}$$

where $K : \mathbb{R}^{2+} \to \mathbb{R}$ is twice continuously differentiable. The integration of (1) yields $q(t) = \int_0^t K(u(s), q(s))ds$, the cumulative level of quality at time t. To simplify presentation, we shall omit the arguments from the functions where there is no confusion.

In line with Li and Rajagopalan (1998), quality dynamics $\dot{q}$ depends on both autonomous learning (there is an autonomous effect of quality q) and induced learning (via the investment in innovation u). Note that to increase modeling tractability, the dynamics of quality does not depend on past production. We will see in the model analysis that this simplifying assumption is without loss of generality. Innovation u increases quality q with diminishing marginal returns, which is consequent with the principle of Pareto (Juran and Gryna, 1988). The formal implication of diminishing returns is that K is increasing concave in u (See Li and Rajagopalan (1998) and Nair and Narasimhan (2006) for similar modeling.):

$$K_u > 0, \; K_{uu} < 0. \tag{2}$$

The model allows for autonomous quality dynamics for which Ittner et al. (2001) provide empirical support. Following Deming (1982), the case $K_q \geqslant 0$ captures autonomous improvement of quality, and any quality improvement is cumulative. The case $K_q < 0$ stands for autonomous deterioration. K_{uq} measures the impact on the dynamics of quality of larger innovation u following better quality q. There is no restriction on the sign of K_{uq}. Because of Schwartz's theorem, we have $K_{uq} = K_{qu}$.

3.2.2 Quality-Based Cost

Production cost is widely acknowledged to depend on product quality (Schiffauerova and Thomson, 2006; Ittner et al., 2001; Vörös, 2006; Li and Ni, 2016; Pan and Li, 2016). We consider here that cost is quality-based. The unitary production cost function $C : \mathbb{R}^+ \to \mathbb{R}^+$ is twice continuously differentiable and increases with quality q as in Caulkins et al. (2015). Therefore the cost is $C = C(q(t))$ with

$$C_q \geqslant 0. \tag{3}$$

The effect of quality on cost is C_q. The independence of cost to quality $C_q = 0$ and the increase of cost with quality $C_q > 0$ describe, for example, the software and hardware industries (Shy, 2001). A more general unit production cost function may account for the well-documented learning effect of production (Ittner et al., 2001). Such a feature would have the advantage of making the model more realistic, but at the cost of a less tractable analysis (Jørgensen and Zaccour, 2012, p. 70).

3.2.3 Advertising-Based Demand

In the literature, there is wide consensus that advertising fosters demand, though the explanations may differ. Indeed, Bagwell (2007, p. 33) recalls that "the overall effect of advertising on primary demand is difficult to determine and appears to vary accross industries." Two main views of advertising, namely the persuasive and the informative views, compete on the explanation of how advertising impacts demand. According to the persuasive view, advertising changes consumer preferences (Marshall, 1890; Chamberlin, 1933; Kaldor, 1950); following the informative view, advertising provides product information (Ozga, 1960; Stigler, 1961; Telser, 1964). Even if for different reasons, each view considers that more advertising implies greater demand.

In this modeling, advertising expense $a(t) \in \mathbb{R}^+$ is a firm decision variable. The demand function $D : \mathbb{R}^{2+} \to \mathbb{R}^+$ is twice continuously differentiable. The demand of consumers D depends jointly on advertising a and quality q, that is $D = D(a(t), q(t))$.

Demand rises with advertising with diminishing marginal returns. Empirical validation of the diminishing returns of advertising is synthesized in the great survey of Bagwell (2007). Demand increases with product quality and the advertising effect is higher for better product quality.

$$D_a > 0,\ D_{aa} < 0,\ D_q \geqslant 0,\ D_{aq} \geqslant 0. \tag{4}$$

The direct effects of advertising and quality on demand are D_a and D_q. The indirect effect of advertising on demand D_{aq} indicates an increasing return phenomenon ($D_{aq} \geqslant 0$). The general demand function that we model places little restriction on the way advertising affects demand. Indeed, this demand function is compatible with the persuasive and informative views (Bagwell, 2007).

3.2.4 Price

The entire demand is satisfied and there is no inventory; demand equals sales and production. The price (or unit product price) P is given by the inverse demand function $P : \mathbb{R}^+ \to \mathbb{R}^+$ that is twice continuously differentiable. Price P depends of the demand D, and $P = P(D(a(t), q(t)))$. The price decreases with the demand

$$P_D \leqslant 0. \tag{5}$$

The case $P_D < 0$ refers to the monopolistic case, where the firm has market power, and the price reduces if the quantity sold increases. The case $P_D = 0$ represents a first approximation of the competitive case, in which the firm has no market power, and the price is given by the market (Schmalensee, 1978).

3.3 Model Analysis

The current profit $\pi(t)$, with values in $\mathbb{R}$, is

$$\pi(t) = [P(D(a(t), q(t))) - C(q(t))]D(a(t), q(t)) - a(t) - u(t).$$

The firm maximizes the intertemporal profit (or present value of total profit stream) by simultaneously finding the optimal trajectories of advertising and innovation over the planning horizon. The firm accounts for the quality dynamics and the discount rate $r \in \mathbb{R}$. Formally, the objective function of the firm is

$$\max_{a(s),\, u(s) \geqslant 0\ \forall s \in [0,T]} \int_0^T e^{-rt}\pi(t)dt,$$
$$\text{subject to } \dot{q}(t) = K(u(t), q(t)) \text{ with } q(0) = q_0.$$

The intertemporal profit maximization problem is solved with the necessary and sufficient optimality conditions of Pontryagin's maximum principle. On this basis, the shadow price (or current-value adjoint variable) $\lambda(t)$ represents the marginal value of quality on the intertemporal profit at t. To the best of our knowledge, the maximization problem above, with with $u(t)$ and $a(t)$ as decision variables, has not been studied before. Only maximization problems with the controls $q(t)$ and $a(t)$ have been investigated. Our modeling strategy offer two different features from the extant literature. First, it enables to explicitly consider the fixed cost of quality (that is innovation expenses $u(t)$) and the variable cost of quality $C(q)$. Since quality affects the cost of the firm in both ways, the model recognizes that distinction. Second, if quality $q(t)$ is a state variable (opposing a control variable), the dynamic advertising rule does not depend on the dynamic innovation rule, simplifying the analysis. In other words, any advertising rule will be independent from any specification of innovation function $K(u,q)$. We now omit function arguments for clarity when there is no confusion. The current-value Hamiltonian H writes

$$H(a,u,q,\lambda) = [P(D(a,q)) - C(q)]D(a,q) - a - u + \lambda K(u,q).$$

The current-value Hamiltonian H sums the current profit $(P-C)D - a - u$ and the future profit λK. As such, H measures the intertemporal profit.

The maximum principle implies the dynamic of the shadow price λ:

$$\dot{\lambda} = r\lambda - H_q = r\lambda - [(P_D D_q - C_q)D + (P-C)D_q + \lambda K_q], \tag{6}$$

with the transversality condition $\lambda(T) = 0$.

Assuming that they exist, we seek interior solutions for advertising and innovation. The monopolist maximizes the intertemporal profit H if and only if a and u satisfy the necessary first-order conditions:

$$H_a = 0 \implies P - C - \frac{1}{D_a} + P_D D = 0, \tag{7a}$$

$$H_u = 0 \implies K_u - \frac{1}{\lambda} = 0. \tag{7b}$$

Let $a_M(u)$ be the advertising rate that satisfies (7a). This advertising level maximizes the intertemporal profit for any level of innovation. In a similar vein, let $u_M(a)$ denotes the innovation rate that satisfies (7b), and it maximizes the intertemporal profit for any level of advertising. The intertemporal profit is maximal when the firm jointly selects the advertising and innovation pair such that $(a_M, u_M) = (a_M(u_M), u_M(a_M))$.

Following (7a) the markup $P - C$ is strictly positive and the firm never sells at loss because $D_a > 0$ from (4) and $P_D \leqslant 0$ from (5). Recall that the shadow price of quality λ corresponds to the marginal impact of quality the intertemporal profit. Consequently larger λ implicates that the firm makes greater profits with better quality. When λ raises, the firm receives incentives to increase quality, and it enhances innovation expenses $u_M(a)$. Note that the impact of additional innovation on quality K_u falls since the diminishing returns of innovation $K_{uu} < 0$ from (2).

For the maximization of the intertemporal profit H, we further assume the three following second-order conditions (concavity of H with respect to a and u):

$$H_{aa} < 0 \implies -D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{(D_a)^2} > 0, \tag{8a}$$

$$H_{uu} < 0 \implies \lambda K_{uu} < 0, \tag{8b}$$

$$H_{aa}H_{uu} - H_{au}^2 > 0. \tag{8c}$$

The detailed proof of the implication of Condition (8a) is in Appendix A.1.

The implication of Condition (8b) together with (2) is

$$\lambda(t) > 0,\ \forall\, t\ \in [0, T), \tag{9}$$

which means that better quality always augments the intertemporal profit.

There is no additional constraint with Condition (8c) which is verified because $H_{aa} < 0$, $H_{uu} < 0$ and $H_{au} = 0$.

3.3.1 Value of $\lambda(t)$

Note $\eta_q \equiv \dfrac{\partial D}{\partial q}\dfrac{q}{D}$ the quality elasticity of demand and $\eta_a \equiv \dfrac{\partial D}{\partial a}\dfrac{a}{D}$ the advertising elasticity of demand. The intertemporal value at time t of a marginal increase in quality q is given by the integration of (6) that yields[1]

$$\lambda(t) = \int_t^T e^{-r(s-t)+\int K_q}\left(\frac{\eta_q}{\eta_a}\frac{a}{q} - C_q D\right) ds, \tag{10}$$

with the notation abuse $\int K_q$ for $\int_{s-t}^{T} K_q(u(\mu), q(\mu))d\mu$.

The shadow price of quality λ is the net result of the effects of advertising adjustment to quality $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q}$ and total cost C_qD. The effect of advertising adjustment to quality increase $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q}$ measures the optimal advertising adjustment to stimulate demand after an increase in quality. This adjustment depends on the demand sensitivity to quality η_q, the demand sensitivity to advertising η_a, and the advertising expenditure by level of quality $\dfrac{a}{q}$. The advertising-quality effect has a positive impact on λ, that is $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q} > 0$, because any increase in quality fosters the demand, and thus the future profits.

The total cost effect C_qD represents the increase of total cost as quality q increases, demand D remaining constant. The total cost effect has a negative impact on λ since $C_q \geqslant 0$ and $D \geqslant 0$. Better quality augments cost, and diminishes future profits. If the marginal impact of quality on cost is null $C_q = 0$, then the total cost effect disappears and only the advertising-adjustment effect remains. But if $C_q > 0$, the shadow price λ

[1]The proof lies in Appendix A.2.

reduces, that is the marginal impact on the intertemporal profit falls. Thus, the firm has less incentives to offer quality, and innovation decreases.

Conditions (9) and (10) imposes

$$\mu\left(\frac{\eta_q}{\eta_a}\frac{a}{q}\right) > \mu\left(C_q D\right), \ \forall t \in [0, T), \tag{11}$$

where $\mu(.)$ is the mean value function.

A positive shadow price of quality λ requires that the mean value of the advertising-quality effect $\frac{\eta_q}{\eta_a}\frac{a}{q}$ dominates the mean value of the total cost effect $C_q D$. This condition is natural because quality rises (innovation u remaining constant), the firm gains more from higher demand than it looses from higher cost: the net result of better quality on profit is positive. As a result, the firm invests in innovation and develops quality such that increase in demand after adjusting advertising is higher than the total cost of quality. This condition makes sense because if improved quality deteriorates profit, then the firm would not innovate to promote quality.

Equations (7*b*) and (10) indicate that both the demand-side, with consumer preferences for advertising and quality, and the supply-side, with the firm capability for cost and quality, determine the product innovation policy u over time. The model takes thus into account two main views on innovation. Innovation is driven by the consumer in the market pull view and by the firm in the technology push view. Both views taken together explain most innovation features (Teng and Thompson, 1996; Adner and Levinthal, 2001; Chenavaz, 2011, 2012).

3.3.2 Variations of $u(t)$

In line with Chenavaz (2012) and Chenavaz (2017), the transversality condition $\lambda(T) = 0$ in (6) and the condition on the shadow price of quality $\lambda(t) > 0$ for all $t \in [0, T)$ in (9) imply that there is $t_1 \in [0, T)$ such that $\dot{\lambda}(t) < 0$ for all $t \in (t_1, T)$. The shadow price λ declines after time t_1. In addition, because $K_u = \frac{1}{\lambda}$ from (7*b*), then $\dot{K}_u = -\frac{\dot{\lambda}}{\lambda^2}$. So, $\operatorname{sgn} \dot{K}_u = -\operatorname{sgn} \dot{\lambda}$ and for all $t \in (t_1, T)$, there is $\dot{K}_u > 0$. Recall that $\dot{K}_u = K_{uu}\dot{u} + K_{uq}\dot{q}$, $K_{uu} < 0$ in (2), and $\operatorname{sgn} K_{uq}$ is unknown. Assume that $\operatorname{sgn} K_{uq} = 0$, as it is the case for an additively separable innovation function like $K = \sqrt{u} - q$. Then, $\operatorname{sgn} \dot{u} = \operatorname{sgn} \dot{\lambda}$. Therefore innovation falls after t_1. Formally:

$$\exists\, t_1 \in [0, T) \mid \dot{u}(t) < 0, \ \forall t \in (t_1, T). \tag{12}$$

Provided that $K_{uq} = 0$ (or when K_{uq} is "sufficiently" low to be approximated by zero.) innovation falls ($\dot{u} < 0$) in the last part of the product life cycle (from t_1 to T). In the case $t_1 = 0$, innovation always falls.

In the first part of the product life cycle though (from $t = 0$ to t_1) innovation may increase or decrease. Note that because of innovation rule (7*b*), the firm always invests in innovation, even at a decreasing rate.

Result (12) links the sensitivity of the consumer to advertising and quality and to the possibility of the firm in quality and cost. In line with Teng and Thompson (1996); Adner and Levinthal (2001); Chenavaz (2011, 2012), the main innovation is achieved at the beginning of the product life cycle. At the beginning, innovation stabilizes the product and develops new features, which interest the consumer. With product maturity, innovation becomes less essential and falls.

3.3.3 Variations of $a(t)$

Equation (7a) provides the static advertising condition. The advertising condition must hold during the whole planning period, on which the firm has an optimal behavior. At the optimum, marginal revenue variations balance marginal cost variations. Such variations also generate variations in advertising and quality. The link between the dynamics of advertising and quality becomes explicit with the differentiation with respect to time of the static advertising condition[2]:

$$\dot{a}\left(-D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{D_a^2}\right) = \dot{q}\left(D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} - C_q\right), \tag{13}$$

which is called the *rule of dynamic advertising*.

The rule of dynamic advertising (13) originates from the properties of demand $D(a, q)$ and cost $C(q)$ functions. Moreover, it establishes structural and analytical (in opposition to parametric and numerical) links between the dynamics on advertising and quality. Because (13) is solely tied to the static advertising condition (7a), it depends neither on the static innovation condition (7b) nor on the quality dynamics (1). In other words, the rule of dynamic advertising is robust to any innovation process, say an exogenous or a stochastic process that would drive quality dynamics.

It would be convenient to express advertising a in terms of quality q. Because advertising and quality are decision and state variables, it is possible to apply the time elimination method (Mulligan and Sala-i Martin, 1991). Let the decision a be a continuously differentiable function of the state q. In this case, $\dfrac{\dot{a}}{\dot{q}}$ simplifies to a_q which directly measures the impact of quality on advertising. Therefore (13) rewrites as

[2] The detailed proof is in Appendix A.3.

$$\underbrace{a_q \left(-D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{D_a^2} \right)}_{\text{Second-order conditions } (+)}$$
$$= \underbrace{\underbrace{D_q(2P_D + P_{DD}D)}_{\text{Quality } (\pm)} \quad \underbrace{+\frac{D_{aq}}{D_a^2}}_{\text{Advertising } (+)}}_{\text{Demand effects}} \quad \underbrace{\underbrace{-\,C_q}_{\text{Cost } (-)}}_{\text{Supply effect}}, \tag{14}$$

which is identified as the *rule of advertising-quality relationship*.

The rule of advertising quality relationship (14) measures the impact of quality on advertising a_q. Because of the second-order condition (8a), on the left-hand side of (14), the second factor $\left(-D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{D_a^2}\right)$ is positive. On the right-hand side, the total impact of quality on advertising depends on the direct quality effect D_q, on the direct and indirect advertising effects D_a and D_{aq}, and on the cost effect C_q. Quality and advertising effects stand for the demand effects, whereas the cost effect measures the supply effect. We detail deeper these four effects hereafter.

- The direct quality effect D_q captures the impact of better quality on demand. This impact sums two potentially competing effects. First, higher quality increases demand, and thus lowers price ($P_D < 0$). This slope effect is negative. Second, if higher demand reduces the price at a decreasing level (if $P_{DD} > 0$), then the curvature effect is positive because the firm takes advantage of larger demand (if $P_{DD}D > 0$). As a result, it is undetermined whether the firm looses more from lower price P than it benefits from higher demand D; the sign of $(2P_D + P_{DD}D)$ is unknown, and the direct quality effect has an ambiguous impact on the advertising-quality relationship.
- The direct advertising effect D_a captures the raise in demand after an advertising increase. If the direct advertising effect is larger, the firm needs to advertise less to reach the same demand; the direct advertising effect has a negative impact on the advertising-quality relationship.
- The indirect advertising effect D_{aq} measures the higher increase of demand following the advertising of a better quality product, reflecting a synergy effect. Because it captures that synergy, the indirect advertising effect has a positive impact on the advertising-quality relationship.
- The cost effect c_q accounts for the impact of an increase in quality on the unit cost. The higher the cost of quality, the less money remains to advertise. Advertising falls with any increase in cost, and the cost effect has a negative impact on the advertising-quality relationship.

Theorem 1. *The relationship between advertising and quality is characterized by*

Cases	Conditions	Results
Case 1	$D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} > C_q$	$a_q > 0$
Case 2	$D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} = C_q$	$a_q = 0$
Case 3	$D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} < C_q$	$a_q < 0$

Proof. Immediate with (14). □

With regard to Theorem 1, the impact of quality on advertising depends on the relative weigh of the demand effects (the quality and advertising effects) and the supply effect (the cost effect).

- Case 1: In this situation, demand effects outweigh the supply effect. Therefore, higher quality involves more advertising, and the consumer may infer better quality from higher advertising. Case 1 corresponds to the efficient market of the informative view, in which there is complementarity between advertising and quality.

- Case 2: Under this scenario, demand effects exactly balance the supply effect. Consequently, better quality does not impact advertising. If quality is unknown, the consumer cannot infer greater quality from more advertising. There is independence between advertising and quality.

- Case 3: In this case, demand effects are dominated by the supply effect. Thus, higher quality yields less advertising, and the consumer may not deduce better quality from larger advertising. Case 3 matches the perverse market of the persuasive view, where there is substitutability between advertising and quality.

From these three cases the model suggests the following implications: The firm substitutes advertising for quality in the marketing mix in Case 3 but not in Cases 1 and 2. Consumers use advertising as a guide of high quality in Case 1 but not in Cases 2 and 3.

So far, we studied the case where the firm has market power, that is $P_D < 0$. Now, we analyze the case where the firm has no market power, namely $P_D = 0$. There are two justifications for this situation. First, $P_D = 0$ represents a first approximation of the monopolistic case by conjecturing that the qualitative structure of the advertising policy holds. This conjecture is explicit in Dockner and Jørgensen (1988) and implicit in Erickson (2009). Second, in the competitive case, $P_D = 0$ relies on the "oligopolists' tendency to substitute non-price for price competition" (Schmalensee, 1978, p. 487). In each case, the assumption $P_D = 0$ is convenient, though restrictive, as it avoids the study of any pricing behavior over time.

Corollary 1. *If the price does not depend on the quantity sold $P_D = 0$, then the relationship between advertising and quality is characterized by*

Cases	Conditions	Results
Case 1	$\frac{D_{aq}}{D_a^2} > C_q$	$a_q > 0$
Case 2	$\frac{D_{aq}}{D_a^2} = C_q$	$a_q = 0$
Case 3	$\frac{D_{aq}}{D_a^2} < C_q$	$a_q < 0$

Proof. Substitute $P_D = 0$ and $P_{DD} = 0$ in (14). □

Corollary 1 also holds in the more general case of $D_q(2P_D + P_{DD}D) = 0$, that is, if the demand effect D_q is low enough to be approximated by 0 or if the slope effect and the curvature effect compensate each other, that is $2P_D + P_{DD}D = 0$.

According to Corollary 1, the impact of quality on advertising is positive if the advertising effects outweigh the cost effect (Case 1). If the advertising effects balance the cost effect, the impact of quality on advertising is null (Case 2). The impact of quality on advertising is negative if the advertising effects are below the cost effect (Case 3).

Remark 1. *Without market power for the firm ($P_D = 0$), the direct effect of quality on demand D_q does not impact the relationship between advertising and quality.*

Proof. Immediate with Corollary 1. □

Remark 2. *If the cost is independent from quality $C_q = 0$, then quality has a positive impact on advertising.*

Proof. Immediate with Corollary 1 and $C_q = 0$. □

If the cost effect C_q vanishes, then Case 1 applies according to Corollary 1, and quality has a positive effect on advertising. This situation characterizes for instance digital goods. Indeed, for digital goods, the marginal cost is often assumed to be null or very low (Shy, 1995, 2001).

Remark 3. *If the demand function is additively separable $D(a, q) = h(a) + l(q)$, then quality has a negative impact on advertising.*

Proof. Consider $D(a, q) = h(a) + l(q)$ that imposes $D_{aq} = 0$. The proof is immediate with Corollary 1 and $D_{aq} = 0$. □

With an additive separable demand function, the indirect advertising effect D_{aq} vanishes. Case 3 from Corollary 1 applies, and quality has a negative effect on advertising. This case is worth noting since much research uses linear demand functions ($D = a + q$), which have the property of additive separability (Tirole, 1988; Shy, 1995). Another example of demand function additively separable is $D = c + a^\gamma + q$, with the parameters $c \geqslant 0$ and $\gamma \in (0, 1)$.

4 Discussion

This paper studies the question of whether a monopolist with a higher quality product engages in more or less advertising than a monopolist with a lower quality product. The main research question is how such a monopolist should optimally structure its advertising and innovation expenditure policies over time, and how the two policies are related to each other. The paper studies this in a dynamic context. The paper's main result is presented in Theorem 1. The model here is not an attempt to formalize Tellis and Fornell (1988)'s conjecture. The paper is nevertheless in the spirit of their works. It proves their conjecture with no need to assume that consumers may be misled about product quality.

The direct advertising and cost effects are in line with Tellis and Fornell (1988), whereas the quality and indirect advertising effects are new insights derived from this article. More generally, the rule of the advertising-quality relationship shows that better quality increases advertising (dominance of the informative view) if the demand effects overcome the supply effect. Alternatively, the greater quality decreases advertising (preeminence of the persuasive view) if the demand effects are lower than the supply effect.

This article proposes a formal model with known quality, the first of its kind, which integrates the opposing informative and persuasive views on advertising in a unifying framework. By considering these classic views jointly, it reconciles different predictions on the advertising-quality relationship in a contingency perspective. This contingency perspective yields a rule indicating the demand and supply conditions for which the relationship is positive (efficient market in the informative view) or negative (perverse market in the persuasive view). As a result, the rule that is proposed finds an original articulation of the demand- and supply-side, shedding new light on the advertising-quality relationship.

4.1 Testable Implications

The results of this work are the basis for a discussion about its testable implications. Directly based on that rule, Theorem 1 states that advertising and quality are complements if the demand effects (or quality and advertising effects on demand) overcome the supply effect (or cost effect) and substitutes if the demand effects are dominated by the supply effect. When the price is constant, Corollary 1 reveals that the quality effect on demand does not work. In this situation, advertising and quality are complements if the advertising effects outweigh the cost effect, and substitutes if the advertising effects are below the cost effect. Figure 1 represents the complement and substitute spaces for advertising and quality, as expressed by Corollary 1. The implications of Corollary 1 is clearer with the features of goods and industries on the supply- and demand-sides.

The supply-side is usefully characterized by low and high cost industries. The cost effect is low in

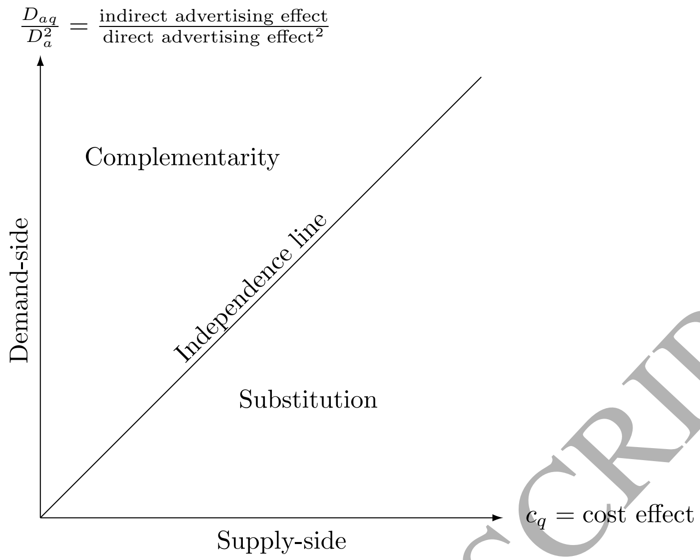

Figure 1: Complement and Substitute Spaces for Advertising and Quality with Respect to Cost and Advertising.

digital industries, because the unit production cost (the cost of information copy or storage) is not larger for a software with more functionalities or for a more entertaining movie. But the cost effect is high for manufacturing industries like traditional or hardware industries. In effect, putting efforts to produce high quality product, such as a more powerful car or a smaller chip, is costly (Shy, 2001). Hence, low cost may denote young or inefficient firms, whereas high cost may account for mature or efficient firms. That is, cost signals efficiency (Bagwell, 2007). According to Corollary 1, advertising and quality are more likely to be complements for low cost, and substitutes for high cost industries or firms.

The value of the new product may be discounted due to the uncertainty associated with it as a result of lack of information. As experience information becomes available, the uncertainty associated with the new product is reduced, and accordingly the product's value increases (Kalish, 1985). In the case of some quality uncertainty, the demand-side is better qualified by Nelson (1970)'s distinction between search and experience goods. The quality of the good is known before purchase for a search good, and after purchase for an experience good. Furniture and jewelry represent classical examples of search goods, whose quality are known before purchase. A movie, for which the consumer knows if he/she likes it after he/she watches it, is an example experience goods.

In this regard, Nelson (1974, p. 734) states that "the advertising for experience qualities is dominantly indirect information and the advertising for search qualities is dominantly direct information." Indeed, the direct advertising effect is lower for search goods for which only information about price and location is needed, but it is greater for experience goods that requires more information to convince the consumer.

In line with Bagwell (2007)'s great synthesis, the indirect advertising effect is larger for experience goods than for search goods and the direct advertising effect is lower for experience goods than for search goods. Consequently, the ratio of advertising effects D_{aq}/D_a^2 is higher for experience goods than for search goods. With regard to Corollary 1, advertising and quality are more likely to be substitutes for search goods, and complements for experience goods.

Summarizing the characterization of industries and goods, Figure 2 presents the following testable implications of Corollary 1. That is, the relationship between the advertising and quality of several products is shown in two by two matrix based on two dimensions- the nature of the goods (i.e., experience and search) and cost (i.e., low and high cost industry). Each of the quadrants implies a different type of relationships. Hence, it can be argued that the relationship between advertising and quality is more likely to be:

- positive for an experience good of a low-cost industry,
- negative for a search good of a high-cost industry,
- mitigate for a search good of a low-cost industry, and
- mitigate for an experience good of a high-cost industry.

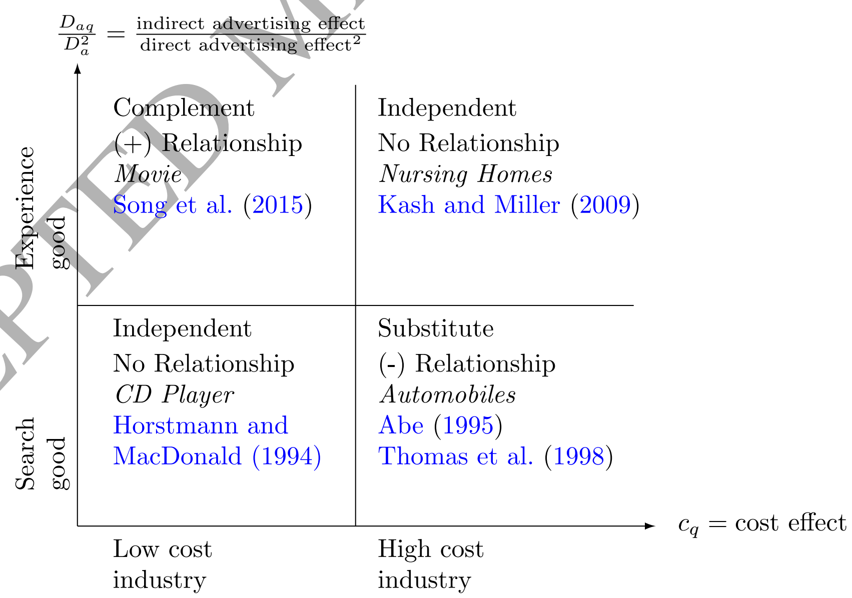

Figure 2: Complement and Substitute Spaces for Advertising and Quality with Respect to Industry and Good.

4.2 Contributions

The main contribution of the present manuscript that builds on a multi-dimensional approach is detailed below. This work provides better understanding of the controversy on this linkage. The paper suggests the conditions that will dictate when the advertising-quality relationship will be positive or negative. Under some conditions, one view may apply, whereas under other conditions, alternative view may be appropriate. The paper provides a novel articulation of prior empirical research by proposing the sign of the advertising-quality relationship being explained by the relative weight of the demand and supply effects. As the spirit of the present article is linked to Tellis and Fornell (1988)'s conjecture, this paper sets up a continuous-time model of a monopolist who faces a world with quality that evolves dynamically as a function of innovation investment (presumably R&D or service) and advertising expenditure. Consumer demand in every instant in turn depends on advertising and quality at that instant. The positioning of the paper is to derive testable implications, so the proposed contribution is in putting some structure on the "it depends" result. It is proposed, for example in the comparison with Dorfman-Steiner, that there are new insights here.

Moreover, the paper goes beyond the insights of Tellis and Fornell (1988). Both articles study the advertising-quality relationship. The main difference is that Tellis and Fornell (1988) formulate a conjecture with empirical support, whereas the present article develops a formal theoretical framework which proves their conjecture as a special case. In this article, the assumptions of unknown quality (the consumer cannot be mistaken) and firm heterogeneity are relaxed. It is to be noted that the current study focuses on a monopolist, for which the cost is implicitly homogeneous. We show that even with known quality and homogeneous cost, Tellis and Fornell (1988)'s conjecture holds. Indeed, a negative advertising-quality relationship may arise if direct advertising and cost effects are sufficiently large (Case 3 of Theorem 1 in the monopoly scenario and Case 3 of Corollary 1 in the competitive scenario). Further, Tellis and Fornell (1988, p. 68) acknowledge the limitations of their modeling of quality, stating that "The major assumption in our theoretical and empirical model is that quality is exogenous and fixed for each business. The assumption of exogenous quality may appear intuitively unreasonable."

The assumption of exogenous and fixed quality is also relaxed in the present work, which considers endogenous and variable quality depending on innovation. At least, they discuss two effects, namely the direct advertising effect and the cost effect, while we highlight two additional effects that are the indirect advertising and the direct quality effects. In a nutshell, Tellis and Fornell (1988) made a conjecture about the possible positive and negative articulation of advertising and quality. In this paper, we prove this conjecture with fewer assumptions and more effects at work.

This article proposes the first formal model with known quality, which integrates the opposing informative and persuasive views on advertising. By considering these classic views jointly, it reconciles their different predictions on the advertising-quality relationship in a contingency perspective. This contingency perspective yields a rule indicating the demand and supply conditions for which the relationship is positive (efficient market in the informative view) or negative (perverse market in the persuasive view). As a result, the rule that we propose propose an original articulation of the demand- and supply-side, shedding new light on the advertising-quality relationship.

This paper analyzes the conditions under which better product quality involves more or less advertising. For this analysis, an optimal control model is developed with general non-linear functions for demand, cost, and innovation. On this basis, a rule of advertising-quality relationship is derived, based on structural and analytical results–as opposed to parameter and numerical results. According to this rule, quality has a positive effect on advertising if the demand effects (quality and advertising effects on demand) overcome the supply effect (quality effect on cost). On the contrary, quality has a negative effect on advertising if the demand effects fall behind the supply effect. This rule represents the first theoretical foundation to both positive and negative relationships between advertising and quality in the simplest modeling.

4.3 Future Research

The modeling simplicity provides a tractable framework offering analytical results for a general demand function. The strength of this simplifying approach is to offer interpretable results with managerial implications about the advertising-quality linkage. Such implications can be empirically tested. In effect, the demand function D (and thus the derivatives D_a, D_q ,and D_{aq}) may be easily estimated, provided availability of dataset on demand (through scanner data for instance). Estimations of D_a is possible following Tellis and Fornell (1988)'s example and estimation of D_{aq} can be made with interaction or moderation variables. Such empirical work is beyond the scope of the present article, and it is left for future work.

The modeling approach also exhibits weaknesses in disregarding some realistic ingredients. Indeed, we assume that the price depends directly only on the demand and neither on the advertising expense nor on product quality, which could affect the willingness to pay. Consequently, the price is given by an implicit demand function and is not decided by the firm. Also, there is no carry over effect of advertising (through goodwill) and sales (via diffusion, saturation, and experience). Eventually, the setup ignores firm competition and demand uncertainty. We omit here such essential elements to focus on the sign of the advertising-quality relationship in the simplest tractable way, without to let other realistic effects play a role. We recognize though that such omissions limit the applicability of the results derived from the model, calling for deeper examination in future research.

5 Conclusion

This paper develops a model of the advertising-quality relationship in an optimal control setting by building on Tellis and Fornell (1988). A distinguishing modification is the inclusion of conditions, which has the effect of increasing the size of the market. These results are consistent with the proposition that many people spontaneously assume high advertising expense implies managerial confidence and high quality. This paper introduces a framework for modeling innovation diffusion that includes price and advertising. A novelty of the model is that it is necessary to consider the joint action of quality and advertising, which, as already discussed, have conflicting effects. Firms can step up quality of their output but, at the same time, increase investment in advertising.

The rule of advertising-quality relationship provides formal guarantee to and expands prior results. More specifically, the rule proves the conjecture of Tellis and Fornell (1988), according to which advertising and quality are substitutes if the direct advertising effects and the cost are high enough. More generally, the rule newly articulates the demand- (through research and experience goods) and supply-sides (via low and high cost industries). This articulation, providing a deeper understanding of the links between advertising and quality, paves the way to theoretical implications that require further research for empirical validation.

A Appendix

A.1 Proof of Equation (8a)

Recalling (7a), the first-order condition with respect to a is:

$$H_a = 0 \implies P - C - \frac{1}{D_a} + P_D D = 0,$$

which is a rearrangement of

$$H_a = 0 \implies P_D D_a D + (P - C) D_a - 1 = 0.$$

Assuming an interior solution, the Hamiltonian is concave in the decision variable a, and the second-order condition with respect to a writes

$$\begin{aligned}
&H_{aa} < 0 \\
&\implies P_{DD} D_a D_a D + D_{aa} P_D D + P_D D_a D_a + P_D D_a D_a + (P - C) D_{aa} < 0, \\
&\implies D_a^2 (2P_D + P_{DD} D) + D_{aa}(P_D D + P - C) < 0.
\end{aligned}$$

Substitute in this result $P - C = \frac{1}{D_a} - P_D D$ from (7a) gives

$$D_a^2 (2P_D + P_{DD} D) + D_{aa}(\frac{1}{D_a}) < 0.$$

Multiply by $-\frac{1}{D_a}$ yields

$$-D_a(2P_D + P_{DD} D) - \frac{D_{aa}}{(D_a)^2} > 0,$$

which completes the proof.

A.2 Proof of Equation (10)

Recall that the dynamic of λ writes in (6)

$$\dot{\lambda} = r\lambda - ((P_D D_q - C_q) D + (P - C) D_q + \lambda K_q), \text{ with } \lambda(T) = 0.$$

Substitute in this result $P - C = \frac{1}{D_a} - P_D D$ from (7a) and rearrange

$$\dot{\lambda} = (r - K_q)\lambda + C_q D - \frac{D_q}{D_a}, \text{ with } \lambda(T) = 0.$$

Recall $\eta_q \equiv \frac{\partial D}{\partial q}\frac{q}{D}$ and $\eta_a \equiv \frac{\partial D}{\partial a}\frac{a}{D}$ and substitute

$$\dot{\lambda} = (r - K_q)\lambda + C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q};\ \lambda(T) = 0.$$

Consider the integrating factor $e^{(-rt+\int K_q)}$, such that

$$\frac{d\lambda(t)e^{(-rt+\int K_q)}}{dt} = e^{(-rt+\int K_q)}(\dot{\lambda} - (r - K_q)\lambda).$$

Since $\dot{\lambda} - (r - K_q)\lambda = C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}$, then

$$\frac{d\lambda(t)e^{(-rt+\int K_q)}}{dt} = e^{(-rt+\int K_q)}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right),$$

and thus

$$d\lambda(t)e^{(-rt+\int K_q)} = e^{(-rt+\int K_q)}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right) dt.$$

Consequently,

$$\int_t^T d\lambda(s)e^{(-rs+\int K_q)} = \int_t^T e^{(-rs+\int K_q)}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right) ds,$$

and

$$\lambda(T)e^{(-rT+\int K_q)} - \lambda(t)e^{(-rt+\int K_q)} = \int_t^T e^{(-rs+\int K_q)}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right) ds.$$

The substitution of the transversality condition $\lambda(T) = 0$ yields

$$\lambda(t) = \int_t^T e^{-r(s-t)+\int K_q}\left(\frac{\eta_q}{\eta_a}\frac{a}{q} - C_q D\right) ds,$$

which completes the proof.

A.3 Proof of Equation (13)

The first-order condition with respect to a (7a) that writes $P - C - \frac{1}{D_a} + P_D D = 0$ is a rearrangement of the immediate condition

$$H_a = 0 \implies P_D D_a D + (P - C)D_a - 1 = 0.$$

Derivate the last condition with respect to t:

$$P_{DD}(D_a\dot{a}+D_q\dot{q})D_aD+P_D(D_{aa}\dot{a}+D_{aq}\dot{q})D+P_DD_a(D_a\dot{a}+D_q\dot{q})$$
$$+P_D(D_a\dot{a}+D_q\dot{q})D_a-C_q\dot{q}D_a+(P-C)(D_{aa}\dot{a}+D_{aq}\dot{q})=0.$$

A rearrangement yields

$$-\dot{a}(D_a^2(2P_D+P_{DD}D)+D_{aa}(P_DD+P-C))$$
$$=\dot{q}(D_q(P_{DD}D_aD+2P_DD_a)+D_{aq}(P_DD+P-C)-C_qD_a)$$

Substitute $P-C=\dfrac{1}{D_a}-P_DD$ from (7a) and divide by D_a:

$$\dot{a}\left(-D_a(2P_D+P_{DD}D)-\frac{D_{aa}}{D_a^2}\right)=\dot{q}\left(D_q(2P_D+P_{DD}D)+\frac{D_{aq}}{D_a^2}-C_q\right),$$

which completes the proof.

References

Abe, M. (1995). Price and advertising strategy of a national brand against its private-label clone: A signaling game approach. *Journal of Business Research 33*(3), 241–250.

Ackerberg, D. A. (2001). Empirically distinguishing informative and prestige effects of advertising. *The RAND Journal of Economics 32*(2), 316–333.

Ackerberg, D. A. (2003). Advertising, learning, and consumer choice in experience good markets: An empirical examination*. *International Economic Review 44*(3), 1007–1040.

Adner, R. and D. Levinthal (2001). Demand heterogeneity and technology evolution: Implications for product and process innovation. *Management Science 47*(5), 611–628.

Anderson, S. P. and R. Renault (2006). Advertising content. *The American Economic Review 96*(1), 93–113.

Archibald, R. B., C. A. Haulman, and C. E. Moody Jr (1983). Quality, price, advertising, and published quality ratings. *Journal of Consumer Research 9*(4), 347–356.

Arrow, K. J. (1962). The economic implications of learning by doing. *The Review of Economic Studies 29*(3), 155–173.

Bagwell, K. (2007). The economic analysis of advertising. *Handbook of Industrial Organization 3*, 1701–1844.

Bagwell, K. and G. Ramey (1993). Advertising as information: Matching products to buyers. *Journal of Economics & Management Strategy 2*(2), 199–243.

Bagwell, K. and G. Ramey (1994). Coordination economies, advertising, and search behavior in retail markets. *The American Economic Review 84*(3), 498–517.

Banting, P. M. and R. E. Ross (1973). The marketing mix: A Canadian perspective. *Journal of the Academy of Marketing Science 1*(1), 1–11.

Caulkins, J. P., G. Feichtinger, D. Grass, R. F. Hartl, P. M. Kort, and A. Seidl (2015). History-dependence generated by the interaction of pricing, advertising and experience quality. Technical Report Research Report 2015-05, Vienna University of Technology.

Caves, R. E. (1986). Information structures of product markets. *Economic Inquiry 24*(2), 195–212.

Caves, R. E. and D. P. Greene (1996). Brands' quality levels, prices, and advertising outlays: Empirical evidence on signals and information costs. *International Journal of Industrial Organization 14*(1), 29–52.

Chamberlin, E. (1933). *The Theory of Monopolistic Competition.* Harvard University Press, Cambridge, Massachusetts.

Chenavaz, R. (2011). Dynamic pricing rule and R&D. *Economics Bulletin 31*(3), 2229–2236.

Chenavaz, R. (2012). Dynamic pricing, product and process innovation. *European Journal of Operational Research 222*(3), 553–557.

Chenavaz, R. (2016). Dynamic quality policies with reference quality effects. *Applied Economics*, 1–7.

Chenavaz, R. (2017). Better product quality may lead to lower product price. *The BE Journal of Theoretical Economics 17*(1).

Chioveanu, I. (2008). Advertising, brand loyalty and pricing. *Games and Economic Behavior 64*(1), 68–80.

Chutani, A. and S. P. Sethi (2012). Optimal advertising and pricing in a dynamic durable goods supply chain. *Journal of Optimization Theory and Applications 154*(2), 615–643.

Colombo, L. and L. Lambertini (2003). Dynamic advertising under vertical product differentiation. *Journal of Optimization Theory and Applications 119*(2), 261–280.

Comanor, W. S. and T. A. Wilson (1979). The effect of advertising on competition: A survey. *Journal of Economic Literature 17*(2), 453–476.

Crosby Philip, B. (1984). *Quality Without Tears: The Art of Hassle-Free Management.* McGraw-Hill, New York.

Dawid, H. and G. Feichtinger (1995). Complex optimal policies in an advertising diffusion model. *Chaos, Solitons & Fractals 5*(1), 45–53.

Deming, W. E. (1982). *Quality, Productivity and Competitive Position.* Cambridge, MA, USA: Massachusetts Institute of Technology Center for Advanced Engineering Studies.

Dockner, E. and S. Jørgensen (1988). Optimal advertising policies for diffusion models of new product innovation in monopolistic situations. *Management Science 34*(1), 119–130.

Doganoglu, T. and D. Klapper (2006). Goodwill and dynamic advertising strategies. *Quantitative Marketing and Economics 4*(1), 5–29.

Dorroh, J. R., T. R. Gulledge, and N. K. Womer (1994). Investment in knowledge: A generalization of learning by experience. *Management Science 40*(8), 947–958.

ACCEPTED MANUSCRIPT

Dutton, J. M. and A. Thomas (1984). Treating progress functions as a managerial opportunity. *Academy of Management Review 9*(2), 235–247.

El Ouardighi, F., G. Feichtinger, D. Grass, R. Hartl, and P. M. Kort (Forthcoming 2016a). Autonomous and advertising-dependent 'word of mouth' under costly dynamic pricing. *European Journal of Operational Research*.

El Ouardighi, F., G. Feichtinger, D. Grass, R. F. Hartl, and P. M. Kort (Forthcoming 2016b). Advertising and quality-dependent word-of-mouth in a contagion sales model. *Journal of Optimization Theory and Applications*.

Erdem, T., M. P. Keane, and B. Sun (2008). A dynamic model of brand choice when price and advertising signal product quality. *Marketing Science 27*(6), 1111–1125.

Erickson, G. M. (2009). An oligopoly model of dynamic advertising competition. *European Journal of Operational Research 197*(1), 374–388.

Feichtinger, G. (1982). Saddle point analysis in a price-advertising model. *Journal of Economic Dynamics and Control 4*, 319–340.

Feichtinger, G. (1992). Hopf bifurcation in an advertising diffusion model. *Journal of Economic Behavior & Organization 17*(3), 401–411.

Feichtinger, G., R. F. Hartl, and S. P. Sethi (1994). Dynamic optimal control models in advertising: Recent developments. *Management Science 40*(2), 195–226.

Fine, C. H. (1986). Quality improvement and learning in productive systems. *Management Science 32*(10), 1301–1315.

Fluet, C. and P. G. Garella (2002). Advertising and prices as signals of quality in a regime of price rivalry. *International Journal of Industrial Organization 20*(7), 907–930.

Fruchter, G. E. and C. Van den Bulte (2011). Why the generalized bass model leads to odd optimal advertising policies. *International Journal of Research in Marketing 28*(3), 218–230.

Grosset, L., P. Roberti, and B. Viscolani (2011). A goodwill model with predatory advertising. *Operations Research Letters 39*(6), 419–422.

Grossman, G. M. and C. Shapiro (1984). Informative advertising with differentiated products. *The Review of Economic Studies 51*(1), 63–81.

Gupta, M. C. and C. A. Di Benedetto (2007). Optimal pricing and advertising strategy for introducing a new business product with threat of competitive entry. *Industrial Marketing Management 36* (4), 540–548.

Helmes, K., R. Schlosser, and M. Weber (2013). Optimal advertising and pricing in a class of general new-product adoption models. *European Journal of Operational Research 229* (2), 433–443.

Hertzendorf, M. N. (1993). I'm not a high-quality firm-but I play one on TV. *The RAND Journal of Economics 24* (2), 236–247.

Hertzendorf, M. N. and P. B. Overgaard (2001). Price competition and advertising signals: Signaling by competing senders. *Journal of Economics & Management Strategy 10* (4), 621–662.

Horstmann, I. J. and G. M. MacDonald (1994). When is advertising a signal of product quality? *Journal of Economics & Management Strategy 3* (3), 561–584.

Horstmann, I. J. and S. Moorthy (2003). Advertising spending and quality for services: The role of capacity. *Quantitative Marketing and Economics 1* (3), 337–365.

Huang, J., M. Leng, and L. Liang (2012). Recent developments in dynamic advertising research. *European Journal of Operational Research 220* (3), 591–609.

Ittner, C. D., V. Nagar, and M. V. Rajan (2001). An empirical examination of dynamic quality-based learning models. *Management Science 47* (4), 563–578.

Jerry, C. O. (1977). *Price as an informational cue: Effects on product evaluation*, pp. 267–286. North Holland Publishing Company New York.

Johnson, J. P. and D. P. Myatt (2006). On the simple economics of advertising, marketing, and product design. *The American Economic Review 93* (3), 756–784.

Jørgensen, S., P. M. Kort, and G. Zaccour (2006). Advertising an event. *Automatica 42* (8), 1349–1355.

Jørgensen, S., P. M. Kort, and G. Zaccour (2009). Optimal pricing and advertising policies for an entertainment event. *Journal of Economic Dynamics and Control 33* (3), 583–596.

Jørgensen, S. and G. Zaccour (2012). *Differential Games in Marketing*, Volume 15. Springer Science & Business Media.

Jørgensen, S. and G. Zaccour (2014). A survey of game-theoretic models of cooperative advertising. *European Journal of Operational Research 237* (1), 1–14.

Juran, J. and F. Gryna (1988). *Juranís Quality Control Handbook*. NY: McGraw-Hill.

Kaldor, N. (1950). The economic aspects of advertising. *The Review of Economic Studies 18*(1), 1–27.

Kalish, S. (1985). A new product adoption model with price, advertising, and uncertainty. *Management science 31*(12), 1569–1585.

Karray, S. and G. Martín-Herrán (2009). A dynamic model for advertising and pricing competition between national and store brands. *European Journal of Operational Research 193*(2), 451–467.

Kash, B. A. and T. R. Miller (2009). The relationship between advertising, price, and nursing home quality. *Health care management review 34*(3), 242–250.

Kihlstrom, R. E. and M. H. Riordan (1984). Advertising as a signal. *The Journal of Political Economy 92*(3), 427–450.

Kirmani, A. (1997). Advertising repetition as a signal of quality: If it's advertised so much, something must be wrong. *Journal of Advertising 26*(3), 77–86.

Kirmani, A. and P. Wright (1989). Money talks: Perceived advertising expense and expected product quality. *Journal of Consumer Research 16*(3), 344–353.

Kotler, P. and K. Keller (2006). *Marketing and Management*. Pearson Prentice Hall, Upper Saddle River, NJ, USA.

Krishnamoorthy, A., A. Prasad, and S. P. Sethi (2010). Optimal pricing and advertising in a durable-good duopoly. *European Journal of Operational Research 200*(2), 486–497.

Krishnan, T. V. and D. C. Jain (2006). Optimal dynamic advertising policy for new products. *Management Science 52*(12), 1957–1969.

Levy, F. K. (1965). Adaptation in the production process. *Management Science 11*(6), B–136.

Li, G. and S. Rajagopalan (1998). Process improvement, quality, and learning effects. *Management Science 44*(11-part-1), 1517–1532.

Li, S. and J. Ni (2016). A dynamic analysis of investment in process and product innovation with learning-by-doing. *Economics Letters Forthcoming*, doi:10.1016/j.econlet.2016.05.031.

Linnemer, L. (2002). Price and advertising as signals of quality when some consumers are informed. *International Journal of Industrial Organization 20*(7), 931–947.

Linnemer, L. (2012). Dissipative advertising signals quality: Static model with a continuum of types. *Economics Letters 114*(2), 150–153.

Marshall, A. (1890). *Principles of Economics.* London: MacMillan and Co.

Meurer, M. and D. O. Stahl II (1994). Informative advertising and product match. *International Journal of Industrial Organization 12*(1), 1–19.

Milgrom, P. and J. Roberts (1986). Price and advertising signals of product quality. *The Journal of Political Economy 94*(4), 796–821.

Moorthy, S. and S. A. Hawkins (2005). Advertising repetition and quality perception. *Journal of Business Research 58*(3), 354–360.

Moorthy, S. and H. Zhao (2000). Advertising spending and perceived quality. *Marketing Letters 11*(3), 221–233.

Moraga-González, J. L. (2000). Quality uncertainty and informative advertising. *International Journal of Industrial Organization 18*(4), 615–640.

Mulligan, C. B. and X. Sala-i Martin (1991). A note on the time-elimination method for solving recursive dynamic economic models. *NBER Working Papers 116.*

Nair, A. and R. Narasimhan (2006). Dynamics of competing with quality-and advertising-based goodwill. *European Journal of Operational Research 175*(1), 462–474.

Narasimhan, R. and S. Ghosh (1994). A dynamic model of manufacturing quality's effect on optimal advertising and pricing policies. *European Journal of Operational Research 72*(3), 485–502.

Nelson, P. (1970). Information and consumer behavior. *The Journal of Political Economy 78*(2), 311–329.

Nelson, P. (1974). Advertising as information. *The Journal of Political Economy 82*(4), 729–754.

Nelson, P. (1975). The economic consequences of advertising. *The Journal of Business 48*(2), 213–241.

Nerlove, M. and K. J. Arrow (1962). Optimal advertising policy under dynamic conditions. *Economica 29*(114), 129–142.

Nichols, M. W. (1998). Advertising and quality in the us market for automobiles. *Southern Economic Journal 64*(4), 922–939.

Orzach, R., P. B. Overgaard, and Y. Tauman (2002). Modest advertising signals strength. *The RAND Journal of Economics 33*(2), 340–358.

Ozga, S. A. (1960). Imperfect markets through lack of knowledge. *The Quarterly Journal of Economics 74*(1), 29–52.

Pan, X. and S. Li (2016). Dynamic optimal control of process-product innovation with learning by doing. *European Journal of Operational Research 248*(1), 136–145.

Piga, C. A. (1998). A dynamic model of advertising and product differentiation. *Review of Industrial Organization 13*(5), 509–522.

Piga, C. A. (2000). Competition in a duopoly with sticky price and advertising. *International Journal of Industrial Organization 18*(4), 595–614.

Schiffauerova, A. and V. Thomson (2006). Managing cost of quality: Insight into industry practice. *The TQM Magazine 18*(5), 542–550.

Schmalensee, R. (1972). *The Economics of Advertising.* North-Holland Publishing Company Amsterdam.

Schmalensee, R. (1978). A model of advertising and product quality. *The Journal of Political Economy 83*(3), 485–503.

Sethi, S. P., A. Prasad, and X. He (2008). Optimal advertising and pricing in a new-product adoption model. *Journal of Optimization Theory and Applications 139*(2), 351–360.

Sheshinski, E. (1967). Tests of the "learning by doing" hypothesis. *The Review of Economics and Statistics 49*(4), 568–578.

Shy, O. (1995). *Industrial Organization: Theory and Applications.* MIT Press.

Shy, O. (2001). *The Economics of Network Industries.* Cambridge University Press.

Song, R., S. Jang, and G. G. Cai (2015). Does advertising indicate product quality? Evidence from prelaunch and postlaunch advertising in the movie industry. *Marketing Letters*, 1–14.

Spence, A. M. (1981). The learning curve and competition. *The Bell Journal of Economics 12*(1), 49–70.

Stigler, G. J. (1961). The economics of information. *The Journal of Political Economy 69*(3), 213–225.

Swami, S. and A. Dutta (2010). Advertising strategies for new product diffusion in emerging markets: Propositions and analysis. *European Journal of Operational Research 204*(3), 648–661.

Tellis, G. J. and C. Fornell (1988). The relationship between advertising and product quality over the product life cycle: A contingency theory. *Journal of Marketing Research 25*(1), 64–71.

Telser, L. G. (1964). Advertising and competition. *The Journal of Political Economy 72*(6), 537–562.

Teng, J.-T. and G. L. Thompson (1996). Optimal strategies for general price-quality decision models of new products with learning production costs. *European Journal of Operational Research 93*(3), 476–489.

Thomas, L., S. Shane, and K. Weigelt (1998). An empirical examination of advertising as a signal of product quality. *Journal of Economic Behavior & Organization 37*(4), 415–430.

Tirole, J. (1988). *The Theory of Industrial Organization*. MIT Press.

Tsui, H.-C. (2012). Advertising, quality, and willingness-to-pay: Experimental examination of signaling theory. *Journal of Economic Psychology 33*(6), 1193–1203.

Vörös, J. (2006). The dynamics of price, quality and productivity improvement decisions. *European Journal of Operational Research 170*(3), 809–823.

Womer, N. K. (1979). Learning curves, production rate, and program costs. *Management Science 25*(4), 312–319.

Yelle, L. E. (1979). The learning curve: Historical review and comprehensive survey. *Decision Sciences 10*(2), 302–328.

Yenipazarli, A. (2015). A road map to new product success: Warranty, advertisement and price. *Annals of Operations Research 226*(1), 669–694.

Zhao, H. (2000). Raising awareness and signaling quality to uninformed consumers: A price-advertising model. *Marketing Science 19*(4), 390–396.